Math 242 — Spring 2023
Topics in Algebra

https://sites.aub.edu.lb/kmakdisi/

Problem set 1, due Thursday, February 16 at the beginning of class

Exercises from Fraleigh:

Section 22, exercises 8, 10, 12, 14, 15, 21, 22.
Section 23, exercises 2, 10, 12, 16, 17, 18, 19, 27, 28, 37.

Additional Exercises (also required):

Exercise A1.1: Show that all irreducible polynomials in $\mathbf{R}[x]$ have degree 1 or 2. What about irreducible polynomials in $\mathbf{C}[x]$?

(You may use the fact that all polynomials in $\mathbf{C}[x]$ have complex roots, and properties of the complex conjugate $z = x + iy \Rightarrow \overline{z} = x - yi$.)

Exercise A1.2: Factor each of the following polynomials in $\mathbf{Z}_5[x]$ into irreducible polynomials:

$$f = x^4 + 1, \qquad g = x^4 + 2, \qquad h = x^4 + 4 \qquad\qquad \text{(all in } \mathbf{Z}_5[x]).$$

Exercise A1.3: a) Show that the polynomial $x^4 + x + 1$ is irreducible in $\mathbf{Z}_2[x]$.

b) Why can you deduce that the polynomial $25x^4 + 30x^3 + 33x + 81$ is irreducible in $\mathbf{Q}[x]$?

Look at, but do not hand in, the following exercises:

Section 22, exercises 17, 20, 24, 25, 26, 27, 29, 30, 31.
Section 23, exercises 29, 30, 31, 34, 35.