Markov Fun: Rebecca

On any particular day Rebecca is either cheerful or gloomy. If she is cheerful today then she will be cheerful tomorrow with probability 0.7. If she is gloomy today then she will be gloomy tomorrow with probability 0.4.

(a) If Rebecca is cheerful (gloomy) today what is the probability that she is gloomy (cheerful) tomorrow?

0.3 (0.6).

(b) Write Rebecca "transition probabilities" in a matrix form.

$$P = \begin{pmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{pmatrix}$$

(I.e., we're modeling Rebecca as a "Markov chain.")

(c) If Rebecca is cheerful on Monday, what is the probability that she will be cheerful on Wednesday?

P{C-W} = P{C-Tu} P{C-W} + P{G-Tu} P{C-W}

$= 0.7\times0.7 + 0.3\times0.6 = \mathbf{0.670}$.

(d) Can you find the probability in (c) using matrix *P*?

$$P^2 = \begin{pmatrix} \textcircled{0.67} & 0.33 \\ 0.66 & 0.34 \end{pmatrix}$$

(e) If Rebecca is cheerful on Monday, what is the probability that she will be cheerful on Thursday?

$$
\begin{aligned}
P\{C\text{-}Th\} &= P\{C\text{-}Tu\}\ P\{C\text{-}W\}\ P\{C\text{-}Th\} \\
&\quad + P\{C\text{-}Tu\}\ P\{G\text{-}W\}\ P\{C\text{-}Th\} \\
&\quad + P\{G\text{-}Tu\}\ P\{C\text{-}W\}\ P\{C\text{-}Th\} \\
&\quad + P\{G\text{-}Tu\}\ P\{G\text{-}W\}\ P\{C\text{-}Th\} \\
&= 0.7\times0.7\times0.7 + 0.7\times0.3\times0.6 + 0.3\times0.6\times0.6 \\
&\quad + 0.3\times0.4\times0.6 = \mathbf{0.667}\ .
\end{aligned}
$$

(f) Can you find the probability in (e) using matrix *P*?

$$P^3 = \begin{pmatrix} \boxed{0.667} & 0.333 \\ 0.666 & 0.334 \end{pmatrix}$$

(g) What is the probability that Rebecca is cheerful on Friday if she is *gloomy* on Monday?

$$P^4 = \begin{pmatrix} 0.667 & 0.333 \\ \boxed{0.667} & 0.333 \end{pmatrix}$$

(h) What is the probability that Rebecca is gloomy next year on Feb. 14, if she is gloomy on this year's Feb. 14?

$$P^{365} = \begin{pmatrix} 0.667 & 0.333 \\ 0.667 & \boxed{0.333} \end{pmatrix}$$

(i) What is the probability that Rebecca is cheerful (gloomy) on a given day?

0.667 (0.333).

These are the "stationary probabilities" of the MC (aka "steady state probabilities or long run fractions of time spent in states).