Airline Network Revenue Management

Lebanon's first private airlines, Fly High Airlines (FHA), operates between three cities, Dubai (DXB), Paris (CDG), and Beirut (BEY) (see Figure 1). FHA utilizes Beirut as a hub and flies one round-trip daily between Beirut and each of the other two cities; see Table 1 for the flight information and capacities of the aircraft assigned to the flights. FHA offers tickets in two fare-classes, 1 and 2. As a result, it offers twelve products (types of ticket). These products correspond to itineraries for six origin-destination (OD) pairs, each offered in two fare-classes; see the first column in Table 2, where each product is denoted by indices *ij*, with *i* denoting the OD pair, and *j* denoting the fare-class.

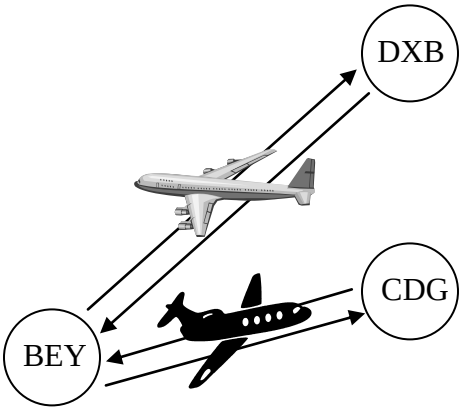

Figure 1 FHA network

Table 1 FHA's flight schedule and capacity assignment

Flight #	**Origin**	**Destination**	**Departs**	**Arrives**	**Capacity**
FH 200	CDG	BEY	8:00	12:30	70
FH 250	BEY	CDG	11:30	15:30	70
FH 300	DXB	BEY	8:00	11:00	50
FH 350	BEY	DXB	13:00	16:00	50

Table 2 FHA's demand forecasts and fares

Product	O	D	Demand		Fares	
			Class	Class 2	Class 1	Class 2
11, 12	CDG	BEY	13	39	$203	$63
21, 22	CDG	DXB	11	28	$303	$93
31, 32	DXB	BEY	14	40	$204	$44
41, 42	DXB	CDG	12	31	$304	$94
51, 52	BEY	DXB	12	39	$203	$53
61, 62	BEY	CDG	11	38	$204	$64

It is February 1 and FHA is determining its capacity control policy for flights on February 15. As of February 1, no bookings have been received for flights on February 15. The demand forecast and fare (price) for each product are given in Table 2. Because of limited capacity FHA cannot satisfy all customer demand. FHA must carefully decide how many tickets of each product to sell.

FHA seeks to determine a "*bid price*" for each product in a way as to accept a reservation if the product fare is at least equal to the bid price.

Formulate FHA problem as a LP.

Develop FHA capacity control policy based on bid prices.

Solution

Let x_{ij} be the number of tickets sold for product ij. FHA network problem can be written as follows:

$$\begin{aligned}
\max \quad & 203x_{11}+63x_{12}+303x_{21}+93x_{22}+204x_{31}+44x_{32}+304x_{41}+94x_{42}+203x_{51}+53x_{52} \\
& +204x_{61}+64x_{62} \\
\text{subject to} \quad & x_{11}+x_{12}+x_{21}+x_{22} \le 70 \qquad (\lambda_{CDG-BEY}) \\
& x_{31}+x_{32}+x_{41}+x_{42} \le 50 \qquad (\lambda_{DXB-BEY}) \\
& x_{41}+x_{42}+x_{61}+x_{62} \le 70 \qquad (\lambda_{BEY-CDG}) \\
& x_{21}+x_{22}+x_{51}+x_{52} \le 50 \qquad (\lambda_{BEY-DXB}) \\
& x_{11} \le 13; \quad x_{12} \le 39 \\
& x_{21} \le 11; \quad x_{22} \le 28 \\
& x_{31} \le 14; \quad x_{32} \le 40 \\
& x_{41} \le 12; \quad x_{42} \le 31 \\
& x_{51} \le 12; \quad x_{52} \le 39 \\
& x_{61} \le 11; \quad x_{62} \le 38 \\
& x_{ij} \ge 0, \; i = 1,\ldots 6, \; j = 1,2 .
\end{aligned}$$

Let $\lambda_{CDG-BEY}$, $\lambda_{DXB-BEY}$, $\lambda_{BEY-CDG}$, and $\lambda_{BEY-DXB}$ be the dual variables corresponding to the capacity constraints for legs CDG-BEY, DXB-BEY, BEY-CDG, and BEY-CDG respectively. Solving LP utilizing Excel solver gives:

$$\begin{aligned}
\lambda_{CDG-BEY} &= \$40 \\
\lambda_{DXB-BEY} &= \$50 \\
\lambda_{BEY-CDG} &= \$44 \\
\lambda_{BEY-DXB} &= \$53
\end{aligned}$$

Utilizing *bid price control*, FHA accepts all reservations where the sum of the corresponding dual variables (also known as *displacement cost*) is less or equal than the corresponding product fare according to Table 3.

Table 3 FHA's capacity control policy

Product	O	D	Displacement Cost	Fares		Accept	
				C 1	C 2	C 1	C 2
11, 12	CDG	BEY	$\lambda_{CDG-BEY} = \$40$	$203	$63	Yes	Yes
21, 22	CDG	DXB	$\lambda_{CDG-BEY} + \lambda_{BEY-DXB} = \$93$	$303	$93	Yes	Yes
31, 32	DXB	BEY	$\lambda_{DXB-BEY} = \$30$	$204	$44	Yes	Yes
41, 42	DXB	CDG	$\lambda_{DXB-BEY} + \lambda_{BEY-CDG} = \$94$	$304	$94	Yes	Yes
51, 52	BEY	DXB	$\lambda_{BEY-DXB} = \$53$	$203	$53	Yes	Yes
61, 62	BEY	CDG	$\lambda_{BEY-CDG} = \$44$	$204	$64	Yes	Yes