

**Example:** (Dual Simplex Method)

$$\begin{aligned} \text{Min } z &= 2x_1 + x_2 \\ \text{s.t.} \quad & 3x_1 + x_2 \geq 3 \\ & 4x_1 + 3x_2 \geq 6 \\ & x_1 + 2x_2 \leq 3 \\ & x_i \geq 0 \end{aligned}$$

$$\Leftrightarrow \begin{aligned} \text{Min } z &= 2x_1 + x_2 \\ \text{s.t.} \quad & -3x_1 - x_2 \leq -3 \\ & -4x_1 - 3x_2 \leq -6 \\ & x_1 + 2x_2 \leq 3 \\ & x_i \geq 0 \end{aligned}$$

$$\Leftrightarrow \begin{aligned} \text{Min } z &= 2x_1 + x_2 \\ \text{s.t.} \quad & -3x_1 - x_2 + S_1 = -3 \\ & -4x_1 - 3x_2 + S_2 = -6 \\ & x_1 + 2x_2 + S_3 = 3 \\ & x_i \geq 0 \end{aligned}$$

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	$x_1$	$x_2$	$S_1$	$S_2$	$S_3$	RHS
<b>Z</b>	-2	-1	0	0	0	0
<b>S<sub>1</sub></b>	-3	-1	1	0	0	-3
<b>S<sub>2</sub></b>	-4	(-3)	0	1	0	-6
<b>S<sub>3</sub></b>	1	2	0	0	1	3
<b>Ratio</b>	1/2	1/3	-	-	-	-

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	$x_1$	$x_2$	$S_1$	$S_2$	$S_3$	RHS
<b>Z</b>	-2/3	0	0	-1/3	0	2
<b>S<sub>1</sub></b>	(-5/3)	0	1	-1/3	0	-1
<b>x<sub>2</sub></b>	4/3	1	0	-1/3	0	2
<b>S<sub>3</sub></b>	-5/3	0	0	2/3	1	-1
<b>Ratio</b>	2/5	-	-	1	-	-

	$x_1$	$x_2$	$S_1$	$S_2$	$S_3$	RHS
<b>Z</b>	0	0	-2/5	-1/5	0	12/5
<b>x<sub>1</sub></b>	1	0	-3/5	1/5	0	3/5
<b>x<sub>2</sub></b>	0	1	4/5	-3/5	0	6/5
<b>S<sub>3</sub></b>	0	0	-1	1	1	0

Then,  $x^*_1 = \frac{3}{5}$ ,  $x^*_2 = \frac{6}{5}$ , and  $Z^* = \frac{12}{5}$ .

Graphically:

