

Example: (Dual Simplex Method)

$$\begin{array}{ll}\text{Min } z = & 2x_1 + x_2 \\ \text{s.t.} & 3x_1 + x_2 \geq 3 \\ & 4x_1 + 3x_2 \geq 6 \\ & x_1 + 2x_2 \leq 3 \\ & x_i \geq 0\end{array}$$

$$\Leftrightarrow \begin{array}{ll}\text{Min } z = & 2x_1 + x_2 \\ \text{s.t.} & -3x_1 - x_2 \leq -3 \\ & -4x_1 - 3x_2 \leq -6 \\ & x_1 + 2x_2 \leq 3 \\ & x_i \geq 0\end{array}$$

$$\Leftrightarrow \begin{array}{llllll}\text{Min } z = & 2x_1 & + & x_2 & & \\ \text{s.t.} & -3x_1 & - & x_2 & + & S_1 & = & -3 \\ & -4x_1 & - & 3x_2 & & + & S_2 & = & -6 \\ & x_1 & + & 2x_2 & & & + & S_3 & = & 3 \\ & x_i & \geq & 0 & & & & & & \end{array}$$

	x_1	x_2	S_1	S_2	S_3	RHS
Z	-2	-1	0	0	0	0
S₁	-3	-1	1	0	0	-3
S₂	-4	(-3)	0	1	0	-6
S₃	1	2	0	0	1	3
Ratio	1/2	1/3	-	-	-	-

	x_1	x_2	S_1	S_2	S_3	RHS
Z	-2/3	0	0	-1/3	0	2
S₁	(-5/3)	0	1	-1/3	0	-1
x₂	4/3	1	0	-1/3	0	2
S₃	-5/3	0	0	2/3	1	-1
Ratio	2/5	-	-	1	-	-

	x_1	x_2	S_1	S_2	S_3	RHS
Z	0	0	-2/5	-1/5	0	12/5
x₁	1	0	-3/5	1/5	0	3/5
x₂	0	1	4/5	-3/5	0	6/5
S₃	0	0	-1	1	1	0

Then, $x_1^* = \frac{3}{5}$, $x_2^* = \frac{6}{5}$, and $Z^* = \frac{12}{5}$.

Graphically:

