Network Revenue Management

Mid East airlines (MEA) is a small regional airline which operates between three cities on the East cost, Boston (BOS), New York (JFK), and Washington DC (IAD) (see figure). MEA utilizes IAD as a hub and flies one round-trip daily between IAD and each of the other two cities. MEA offers tickets in two fare classes, Business (Y) and Economy (M). The flights and plane capacities of MEA are summarized in the following table.

MEA schedule and planes capacity

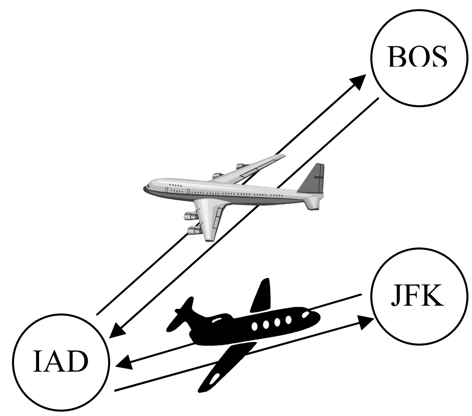
Flight	Origin	Destination	Departs	Arrives	Capacity
200	JFK	IAD	8:00 AM	9:30 AM	67
250	IAD	JFK	11:00	12:30	67
300	BOS	IAD	8:00 AM	10:00	192
350	IAD	BOS	11:30	1:30 PM	192

Suppose that that it's December 7, 2005 and MEA is receiving reservations for its flights on December 14, 2005. MEA problem is to decide which reservation to accept on December 7. MEA has decided on the price and estimates (forecasts) of demand for each of its class and itineraries for the remaining week. The table below describes MEA current status. Product ij with $j = 1$, is a business fare, and with $j = 2$ is an economy fare.

MEA reservations and future demand status on December 7

Product	Orig.	Dest.	Reservations		Demand		Fares	
			Y	M	Y	M	Y	M
11, 12	JFK	IAD	3	20	4	9	$203	$63
21, 22	JFK	BOS	4	26	5	12	$407	$147
31, 32	BOS	IAD	15	99	20	50	$203	$43
41, 42	BOS	JFK	5	25	10	30	$407	$157
51, 52	IAD	BOS	26	102	20	70	$204	$53
61, 62	IAD	JFK	2	13	5	15	$204	$64

MEA seeks to determine a "*bid price*" for each product in a way as to accept reservation only if product fare is above bid price.



MEA Network

Solution

Let x_{ij} be the number of tickets sold for product *ij*. MEA network problem can be written as follows:

$$\begin{aligned}
\max\quad & 203x_{11} + 63x_{12} + 407x_{21} + 147x_{22} + 203x_{31} + 43x_{32} + 407x_{41} + 157x_{42} + 204x_{51} + 53x_{52} \\
& + 204x_{61} + 64x_{62} \\
\text{subject to}\quad & x_{11} + x_{12} + x_{21} + x_{22} \le 14 \qquad (\lambda_{JFk-IAD}) \\
& x_{31} + x_{32} + x_{41} + x_{42} \le 48 \qquad (\lambda_{BOS-IAD}) \\
& x_{41} + x_{42} + x_{61} + x_{62} \le 22 \qquad (\lambda_{IAD-JFK}) \\
& x_{21} + x_{22} + x_{51} + x_{52} \le 34 \qquad (\lambda_{IAD-BOS}) \\
& x_{11} \le 4; \quad x_{12} \le 9 \\
& x_{21} \le 5; \quad x_{22} \le 12 \\
& x_{31} \le 20; \quad x_{32} \le 50 \\
& x_{41} \le 10; \quad x_{42} \le 30 \\
& x_{51} \le 20; \quad x_{52} \le 70 \\
& x_{61} \le 5; \quad x_{62} \le 15 \\
& x_{ij} \ge 0,\ i = 1,\dots 6,\ j = 1, 2
\end{aligned}$$

The right hand side of the first four capacity constraint is obtained by subtracting the reservations on a given leg from the leg Capacity. This gives the remaining leg capacity on December 7. For example, on the leg JFK-IAD there are 23 reservations for the direct flight from JFK to IAD and 30 reservations for the flight from JFK-BOS connecting at IAD. This yields a remaining capacity of 14 on the leg JFK-IAD.

Solving the above utilizing AMPL, gives:

$$\lambda_{JFk-IAD} = \$0$$
$$\lambda_{BOS-IAD} = \$0$$
$$\lambda_{IAD-JFK} = \$157$$
$$\lambda_{IAD-BOS} = \$53$$

Therefore, utilizing *bid price control*, MEA would accept all reservations according to the following table. In the table below, MEA accepts reservations for a product for which the fare exceeds the shadow price (also called displacement cost).

Control Policy on December 7

Product	Orig.	Dest	Displacement Cost	Fares		Accept	
				Y	M	Y	M
11, 12	JFK	IAD	$\lambda_{JFK-IAD} = \$0$	\$20	\$63	Yes	Yes
21, 22	JFK	BOS	$\lambda_{JFK-IAD} + \lambda_{IAD-BOS} = \$53$	\$40	\$14	Yes	Yes
31, 32	BOS	IAD	$\lambda_{BOS-IAD} = \$0$	\$20	\$43	Yes	Yes
41, 42	BOS	JFK	$\lambda_{BOS-IAD} + \lambda_{IAD-JFK} = \$157$	\$40	\$15	Yes	No
51, 52	IAD	BOS	$\lambda_{IAD-BOS} = \$53$	\$20	\$53	Yes	No
61, 62	IAD	JFK	$\lambda_{IAD-JFk} = \$157$	\$20	\$64	Yes	No