

## Chapter 18 Sensitivity Analysis and Expected Value Decisions

- **Sensitivity analysis**

- Engineering economy estimates of parameters such as costs and other cash flow are only an approximation of reality.
- The *realized* future value of a parameter will be generally different from its estimated value.
- Sensitivity analysis attempts to measure the effect of this *uncertainty* (variability) in parameters estimates.
- Sensitivity analysis identifies parameters that have the most impact on an economic decision.
- That is, it identifies the parameters with the highest *sensitivity*.
- Sensitivity of a parameter is the effect of changing the parameter value on the economic criteria such as PW value.

- **Graphical sensitivity analysis**

- Sensitivity analysis is often done graphically by plotting the economic criteria as a function of a parameter.
- The plot is over the range where the parameter is most likely to vary.
- A flat plot indicates insensitivity.

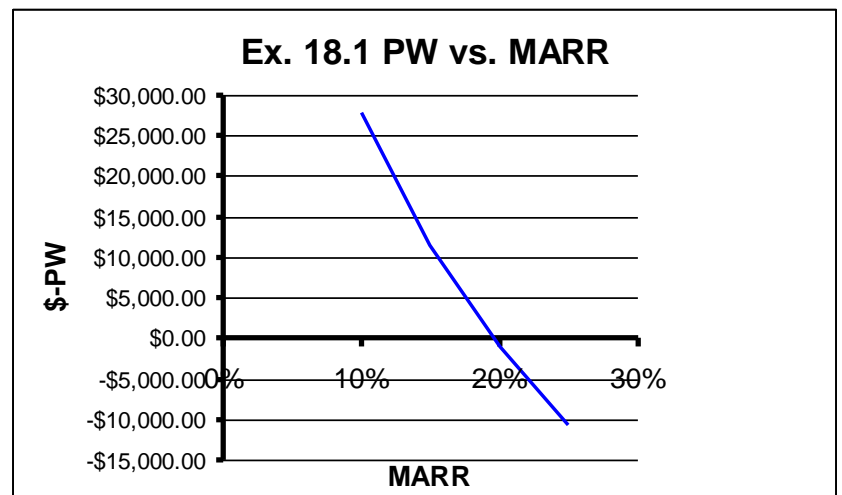
- That is, the parameter has little effect on the economic decision. No need to get very precise estimates for its value.
- A highly variable plot indicates high sensitivity.
- That is, the parameter has a significant impact on economic decision. Estimating its value should be handled with care.

- **Example**

- Example 18.1 gives PW vs. MARR for a project.

Year	Cflow
0	-\$80,000
1	\$25,000
2	\$23,000
3	\$21,000
4	\$19,000
5	\$17,000
6	\$15,000
7	\$13,000
8	\$11,000
9	\$9,000
10	\$7,000

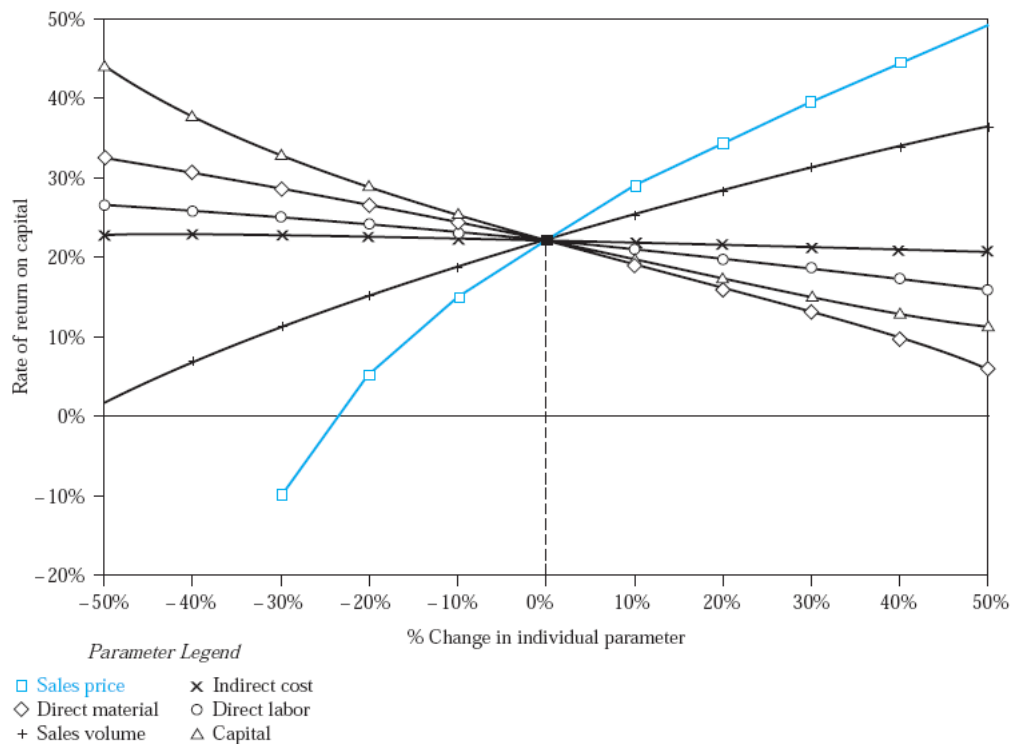
MARR	PW(i%)
10%	\$27,831.49
15%	\$11,510.26
20%	-\$962.36
25%	-\$10,711.51



- In this example, PW is highly sensitive to MARR.

- **Sensitivity analysis with several parameters**

- When several parameters are being analyzed for sensitivity, A *spider plot* can be used.
- This is a plot of the economic criteria as a function of percent changes from the most likely estimates of parameters.
- Example of spider plot (Fig 18.3).



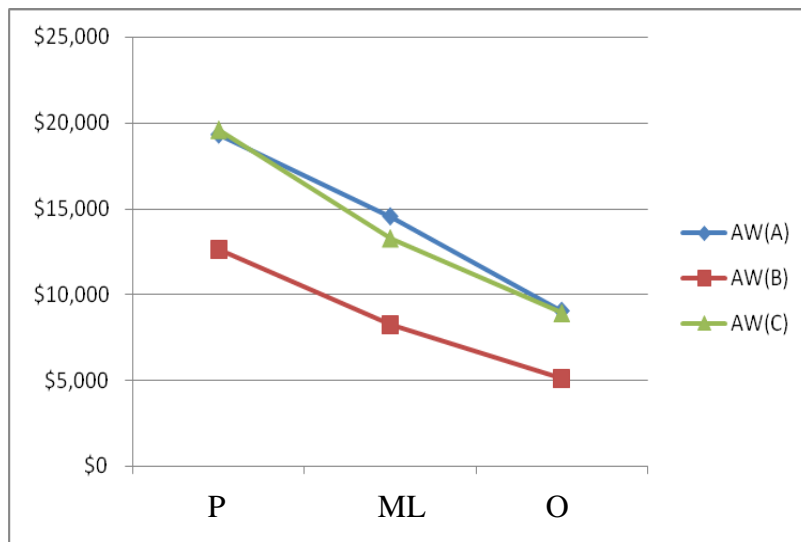
- In this example, ROR is
  - Insensitive to indirect cost (flat curve), and labor cost;
  - Moderately sensitive to material cost and capital;
  - Highly sensitive to sales volume and sales price.

- **Sensitivity analysis with three estimates**

- A common approach is to base sensitivity analysis on three estimates for a parameter: *pessimistic*, *most likely*, and *optimistic* estimate.
- Example 18.3 performs sensitivity analysis of annual cost for three alternatives based on three estimates of parameters.
- In this example, parameters are assumed to change together for the pessimistic, most-likely and optimistic scenarios.
- This is also known as *scenario analysis*.

Strategy	First Cost	SV	AOC	Life	AW
Alternative A					
Pessimistic	-\$20,000	\$0	-\$11,000	3	-\$19,327
Most likely	-\$20,000	\$0	-\$9,000	5	-\$14,548
Optimistic	-\$20,000	\$0	-\$5,000	8	-\$9,026
Alternative B					
Pessimistic	-15,000	500	-4,000	2	-\$12,640
Most likely	-15,000	1,000	-3,500	4	-\$8,229
Optimistic	-15,000	2,000	-2,000	7	-\$5,089
Alternative C					
Pessimistic	-30,000	3,000	-8,000	3	-\$19,601
Most likely	-30,000	3,000	-7,000	7	-\$13,276
Optimistic	-30,000	3,000	-3,500	9	-\$8,927

- According to the plot below, alternative B is better than alternatives A and C under all scenarios.
- Selecting Alternative B is a good choice.



- **Variability and probability**

- When a cash flow (parameter) estimate is highly uncertain and the economic decision is highly sensitive to it, a *probabilistic* cash flow analysis is useful.
- Probability of an event (here the cash flow taken on a value) is typically defined as the long run fraction of time where the event happens.
- If the “event” repeats several times, then one can use historical data to estimate the probability. That is, probability is estimated as a *frequency*.
- E.g., one can estimate that the probability that the energy cost in August exceeds \$1,000 is 0.2, if this cost exceeds 1,000 in 20% of past years.

- However, in several situations, especially in one-time engineering projects, not enough historical data is available.
  - E.g., for the construction of an exotic tower, the experienced civil engineer estimates that there is a 95% chance (i.e. a 0.95 probability) that the cost will not exceed \$1 billion.
  - This is a *subjective* estimate of probability.
- **Cash flows as random variables**
    - A random variable is a function that assigns real numbers to events (outcomes of an experiment).
    - E.g., the salvage value of a machine,  $S$ , is \$1,500 if the market goes up (with probability 0.4) and \$1,000 if the market goes down (with probability 0.6).
    - Then,  $S$  is a random variable with the following *probability distribution*  $P\{S = 1500\} = 0.4$  and  $P\{S = 1000\} = 0.6$ .
    - If the random variable can take on a limited number of values. Then, this is a *discrete random variable*. E.g., the salvage value of the machine above.
    - If the random variable can take on an uncountable number of values. Then, this is a *continuous random variable*.
    - E.g., you may estimate that the monthly fuel consumption of your car is equal likely to be between \$200 and \$250. Then, this consumption is a continuous random variable.

- **Expectation of a random variable**

- The *expectation* of a discrete random variable  $X$  is

$$E[X] = \sum_{x_i} x_i P\{X = x_i\}$$

- E.g., for the machine above the expected salvage value is

$$E[S] = 1,500 \times 0.4 + 1,000 \times 0.6 = \$1,200.$$

- The expectation of a random variable is the value obtained if the underlying experience is repeated for a number of times which is large enough and the resulting values are averaged.

- **Using expected value – Replacing cash flows with their averages**

- Here one finds the expected value of an uncertain (random) cash flow, and then uses this value in the economic analysis, proceeding just like what we have been doing.
- This, often used, approach is only an approximation. It is subject to the *flow of averages*.

- **Example of the flow of average**

- E.g., if the weekly demand for an item selling at \$5 is equally likely to be 100 or 150. The stock level for this item at the beginning of the week is set at 125 units.
- The average demand in this case is 125.
- Replacing the demand by its average, the “expected revenue” is  $125 \times 5 = \$625$ .

- Note however, that there is a 50% chance that the demand will be 100, and the revenue will be  $100 \times 5 = \$500$ .
- There is also a 50% chance that demand will be 150 which exceeds the stock level.
- Then, the revenue when demand is 150 is  $125 \times 5 = \$625$ .
- So, the actual expected revenue is

$$500 \times 0.5 + 625 \times 0.5 = \$562.50.$$

- **Using expected value – Treating cash flows as random variables and finding the expected PW**

- Due to the flow of averages, a better approach for dealing with uncertainty in cash flows is to treat these as random variables and find the expected value,  $E[PW]$  of the PW.
- (This approach is more accurate but it often involves an involved use of *probability theory*. In INDE 303, you can learn more about this.)
- Then, an alternative is considered economically viable if its  $E[PW] \geq 0$ .
- When comparing several alternatives, the alternative with the highest  $E[PW]$  is selected.

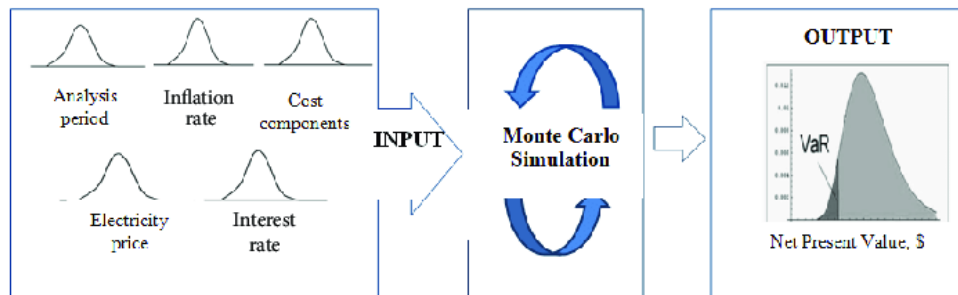


- **Measures of Variability and Analysis beyond E[PW]**

- The *variance* of a discrete random variable  $X$  is

$$\text{Var}[X] = E[(X - E[X])^2] = \sum_{x_i} x_i^2 P\{X = x_i\} - (E[X])^2.$$

- The *standard deviation* of  $X$  is  $\sigma[X] = \sqrt{\text{Var}[X]}$ .
- It measures the variability around the expectation.
- An alternative (or an estimate) having a PW with a large variance should be handled with care.
- In modern risk analysis, a common approach is to use *Monte Carlo simulation* to estimate the full probability distribution of PW based on the distribution of input parameters.



- There are highly user-friendly tools, Excel add-in tools, for doing the Monte Carlo analysis, e.g. @Risk and Crystal Ball.
- In addition, to  $E[PW]$  and  $\text{var}[PW]$ , Monte Carlo simulation allows estimating the *Value at Risk* (VaR), which is the  $\alpha$ -percentile of the distribution.
- Often,  $\alpha = 5\%$ , and the VaR satisfies  $P\{PW \leq \text{VaR}\} = 5\%$ .
- Having a PW with  $\text{VaR} > 0$  is re-assuring, as it implies that there is only a 5% that a project will be economically invalid.