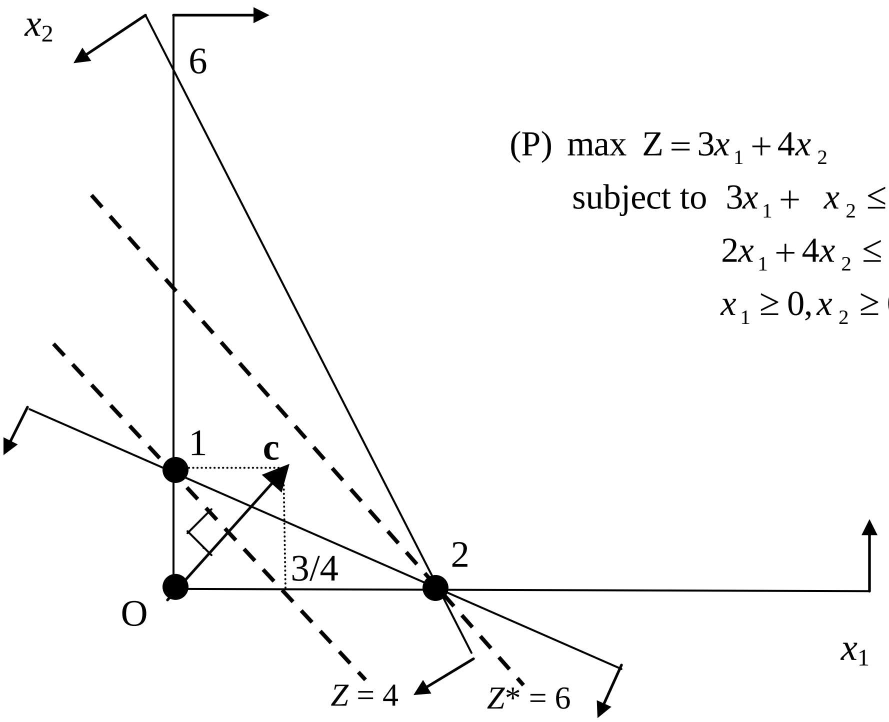
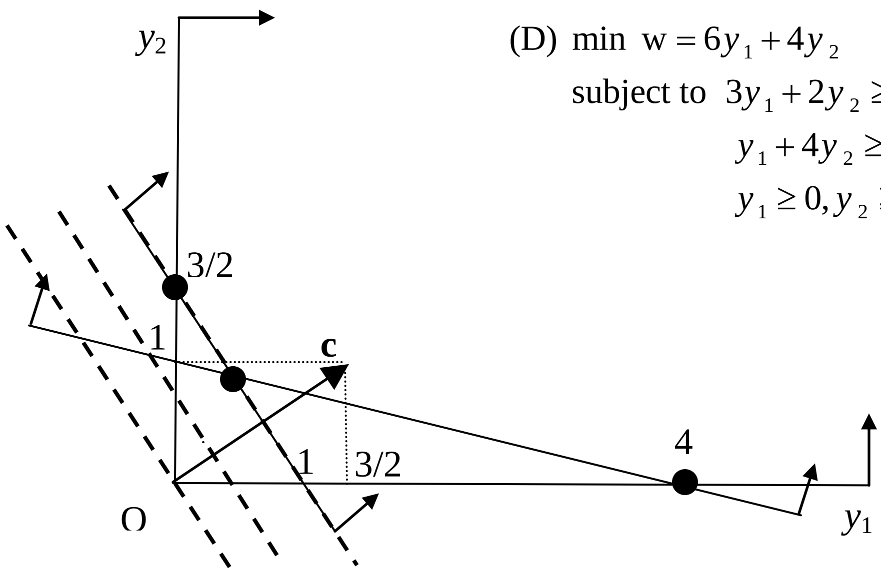
- **Example of primal-dual relationship**
  - Consider the BM company problem.



$$
\begin{aligned}
\text{(P)}\ \ & \max\ \ Z = 3x_1 + 4x_2 \\
& \text{subject to}\ \ 3x_1 + \ \ x_2 \le 6 \quad \leftarrow y_1 \\
& \qquad\qquad\quad 2x_1 + 4x_2 \le 4 \quad \leftarrow y_2 \\
& \qquad\qquad\quad x_1 \ge 0, x_2 \ge 0
\end{aligned}
$$

  - The optimal solution is $x_1^* = 2$, $x_2^* = 0$, and $Z^* = 6$.
  - The dual problem is



$$
\begin{aligned}
\text{(D)}\ \ & \min\ \ w = 6y_1 + 4y_2 \\
& \text{subject to}\ \ 3y_1 + 2y_2 \ge 3 \quad \leftarrow x_1 \\
& \qquad\qquad\quad y_1 + 4y_2 \ge 4 \quad \leftarrow x_2 \\
& \qquad\qquad\quad y_1 \ge 0, y_2 \ge 0
\end{aligned}
$$

  - An optimal solution is $y_1^* = 0$, $y_2^* = 3/2$, and $w^* = 6$.

- To see weak duality in action, pick two feasible solutions of (P) and (D). e.g.,
  - $\mathbf{x}^0 = (0, 1)$ and $\mathbf{y}^0 = (4, 0) \Rightarrow Z(\mathbf{x}^0) = 4 < w(\mathbf{y}^0) = 24$,
  - $\mathbf{x}^1 = (0, 1)$, $\mathbf{y}^1 = (3/2, 1) \Rightarrow Z(\mathbf{x}^1) = 4 < w(\mathbf{y}^1) = 13$,
  - $\mathbf{x}^2 = (1/2, 3/4)$, $\mathbf{y}^2 = (3/2, 1) \Rightarrow Z(\mathbf{x}^2) = 9/2 < w(\mathbf{y}^2) = 13$,
  - and much more. Try it!
- To see strong duality, note that in this case both (P) and (D) have optimal solutions.
- Two optimal solutions of (P) and (D) are $\mathbf{x}^* = (2, 0)$ and $\mathbf{y}^* = (0, 3/2)$. Note that $Z(\mathbf{x}^*) = w(\mathbf{y}^*) = 6$.
- To see complementary slackness, observe that

$$(3x_1^* + x_2^* - 6)y_1^* = (6 + 0 - 6) \times 0 = 0,$$
$$(2x_1^* + 4x_2^* - 4)y_2^* = (4 + 0 - 4) \times (3/2) = 0,$$
$$(3y_1^* + 2y_2^* - 3)x_1^* = (0 + 3 - 3) \times 2 = 0,$$
$$(y_1^* + 4y_2^* - 4)x_2^* = (0 + 6 - 4) \times 0 = 0.$$

- Try it. Pick another optimal solution of (D) and observe that **x*** and this other **y*** satisfy complementary slackness.