

• **A complete example of the simplex method in tabular form**

➤ Consider the LP

$$\begin{aligned} \max \quad & Z = 3x_1 + 2x_2 + 5x_3 \\ \text{s.t.} \quad & x_1 + 2x_2 + x_3 \leq 430 \\ & 3x_1 + \quad + 2x_3 \leq 460 \\ & x_1 + 4x_2 \leq 420 \\ & x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \end{aligned}$$

➤ Standard form

$$\begin{aligned} \max \quad & Z = 3x_1 + 2x_2 + 5x_3 \\ \text{s.t.} \quad & x_1 + 2x_2 + x_3 + S_1 = 430 \\ & 3x_1 + \quad + 2x_3 + S_2 = 460 \\ & x_1 + 4x_2 + S_3 = 420 \\ & x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \end{aligned}$$

➤ All constraints are “ $\leq$ ” and RHS  $> 0$ . Then, the starting solution is O.

|       | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ | RHS | Ratio    |
|-------|-------|-------|-------|-------|-------|-------|-----|----------|
| Z     | -3    | -2    | -5    | 0     | 0     | 0     | 0   | --       |
| $S_1$ | 1     | 2     | 1     | 1     | 0     | 0     | 430 | 430      |
| $S_2$ | 3     | 0     | (2)   | 0     | 1     | 0     | 460 | 230      |
| $S_3$ | 1     | 4     | 0     | 0     | 0     | 1     | 420 | $\infty$ |

|       | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ | RHS  | Ratio    |
|-------|-------|-------|-------|-------|-------|-------|------|----------|
| Z     | 4.5   | -2    | 0     | 0     | 2.5   | 0     | 1150 | --       |
| $S_1$ | -0.5  | (2)   | 0     | 1     | -0.5  | 0     | 200  | 100      |
| $x_3$ | 1.5   | 0     | 1     | 0     | 0.5   | 0     | 230  | $\infty$ |
| $S_3$ | 1     | 4     | 0     | 0     | 0     | 1     | 420  | 105      |

|       | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ | RHS  |
|-------|-------|-------|-------|-------|-------|-------|------|
| Z     | 4     | 0     | 0     | 1     | 2     | 0     | 1350 |
| $x_2$ | -0.25 | 1     | 0     | 0.5   | -0.25 | 0     | 100  |
| $x_3$ | 1.5   | 0     | 1     | 0     | 0.5   | 0     | 230  |
| $S_3$ | 2     | 0     | 0     | -2    | 1     | 1     | 20   |

➤ The optimal solution is  $x_1^* = 0$ ,  $x_2^* = 100$ ,  $x_3^* = 230$  &  $Z^* = 1350$ .