Inventory Theory (2)

- **Continuous review (s, Q) model with probabilistic demand**
  - Consider the following inventory system with probabilistic demand.
  - The inventory level is reviewed continuously.
  - The order policy is as follows: Whenever the inventory level drops below the reorder point, s, place an order of size Q, which arrives after a lead time L.
  - No more than one order can be outstanding.
  - The cost structure is similar to that of the EOQ model, with a holding cost proportional to average inventory at a rate of h (\$/unit/unit time) and an ordering cost of the form $C(x) = K+cx$, when the order quantity $x > 0$.
  - Shortages may occur and excess demand is backordered.
  - The shortage cost is proportional to the number of units short, and incur a one-time cost at a rate of b (\$/unit).
  - Determining an exact optimal policy (i.e. optimal values for s and Q) is complex.
  - In the following we discuss two approximate approaches for obtaining near-optimal policies.

- **Near-optimal (*s*, *Q*) policy based on EOQ and service level**
  - This approach requires knowing the distribution of lead time demand, D_L .
  - The order quantity Q is set equal to that of the EOQ model,

$$Q = \sqrt{\frac{2KE[D]}{h}} ,$$

    where D is the demand per unit time.
  - The reorder point s is determined based on a service level constraint which guarantees that the probability of stock-out is "small", as follows

$$P\{D_L \geq s\} \leq \alpha ,$$

    where $0 < \alpha < 1$.
  - The reorder point can be written as $s = SS + \mu_L$, where SS is the "safety stock", and $\mu_L = E[D_L]$. Then, the problem reduces to determining the safety stock, SS.
  - If demand during lead time is normal with mean and standard deviation μ_L and σ_L, SS is determined as follows.

$$\begin{aligned}
&P\{D_L \geq SS + \mu_L\} \leq \alpha \Rightarrow P\{Z \geq SS / \sigma_L\} \leq \alpha \\
&\Rightarrow P\{Z \leq SS / \sigma_L\} \geq 1 - \alpha \\
&\Rightarrow SS \geq \sigma_L \Phi^{-1}(1 - \alpha),
\end{aligned}$$

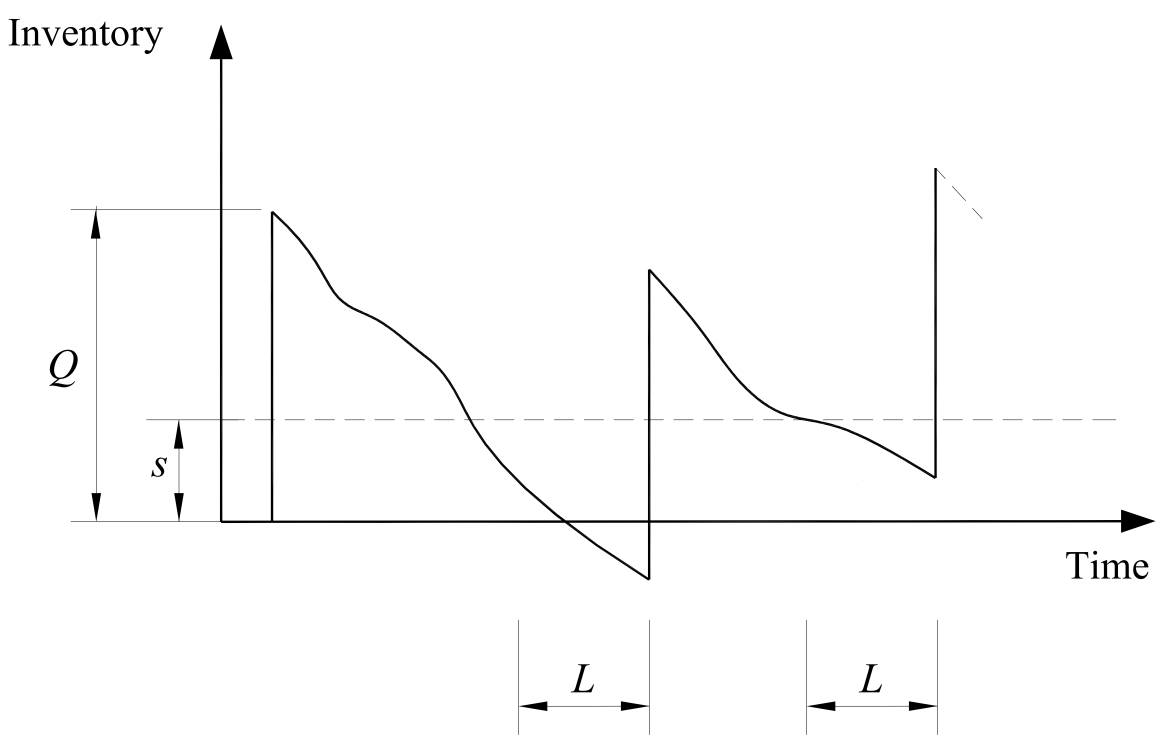
    where $\Phi^{-1}(.)$ is the inverse of the standard normal cdf

- Note that $\Phi^{-1}(1-\alpha)$ can be found from the following table or by using the NORMSINV($1-\alpha$) function in Excel, or from a standard normal table such as Table B.1 (text).

α	0.100	0.075	0.050	0.025	0.010	0.005	0.001
$\Phi^{-1}(1-\alpha)$	1.282	1.440	1.645	1.960	2.326	2.576	3.090

- **Example 6.**
  - Suppose that the three-ohm resistor in Example 1 actually had a probabilistic annual demand which is normally distributed with mean 2,400 units and standard deviation 240 units. Suppose also that the lead time is 1 month.
  - Derive a near-optimal (s, Q) policy with a stock-out probability below 5% .
  - The order quantity is found similar to the EOQ case as $Q = 400$.
  - The mean and standard deviation of leas time demand are $\mu_L = 2400/12 = 200$, and $\sigma_L = 240/12 = 20$.
  - The safety stock level is $SS \geq \sigma_L\Phi^{-1}(1-\alpha) = 20\times1.645 = 32.9$, Set $SS = 33$.
  - The reorder level is $s = SS + \mu_L = 233$.
  - The optimal policy in this case is as follows:
    When inventory drops below 233, place an order for 400.

- **Near-optimal (s, Q) based on approximate holding cost**
  - ➢ This approach also requires knowing the distribution of lead time demand, D_L and the mean demand per unit time $E[D]$.
  - ➢ This approach works by minimizing an approximation for the expected cost per unit time.
  - ➢ The ordering cost is given similar to the EOQ model as $(K + cQ)(\mathrm{E}[D]/Q) = KE[D]/Q + c\mathrm{E}[D]$.
  - ➢ The expected holding cost per unit time is approximated as follows.



  - ➢ The expected inventory level at the end of an ordering cycle is approximately is $s - E[D_L]$.
  - ➢ The expected inventory level at the beginning of an ordering cycle is approximately $s - E[D_L] + Q$.

- The approximation here is ignoring the possibility of shortages at the end of the cycle.
- This is a good approximation if shortages occur sporadically (e.g. when the shortage cost is too high).
- Then, the approximate average inventory level is

$$[(s - E[D_L] + Q) + (s - E[D_L])] / 2 = Q/2 + s - E[D_L] .$$

- The approximate holding cost per unit time is

$$h(Q/2 + s - E[D_L]) .$$

- The shortage cost per cycle is assumed to be proportional to the number of units short. By conditioning on D_L this cost is

$$b\int_s^\infty (x_L - s) f_{D_L}(x_L)dx_L ,$$

where $f_{DL}(.)$ is the pdf of D_L .
- The expected shortage cost per unit time is

$$\frac{E[D]}{Q} b\int_s^\infty (x_L - s) f_{D_L}(x_L)dx_L .$$

- Finally, the expected cost per unit time is

$$EC_U(s,Q) \approx \frac{KE[D]}{Q} + cE[D] + h\left(\frac{Q}{2} + s - E[D_L]\right)$$
$$+ \frac{bE[D]}{Q}\int_s^\infty (x_L - s) f_{D_L}(x_L)dx_L .$$

- It can be shown that $EC_U(s, Q)$ is jointly convex in s and Q.

- The "optimal" values of Q and s can then be obtained by differentiating and setting first derivatives equal to zero.
- Differentiating with respect to Q gives

$$\frac{\partial EC_U(s,Q)}{\partial Q} = -\frac{E[D]}{Q^2}\left(K + b\int_s^{\infty}(x_L - s) f_{D_L}(x_L)dx_L \right) + h\frac{Q}{2} = 0$$

$$\Rightarrow Q^* = \sqrt{\frac{2E[D]\left(K + b\int_{s^*}^{\infty}(x_L - s) f_{D_L}(x_L)dx_L \right)}{h}}$$

- Differentiating with respect to s requires using *Leibniz rule*,

$$\frac{d}{ds}\int_{a_1(s)}^{a_2(s)} f(x,s)dx = \int_{a_1(s)}^{a_2(s)} \frac{\partial f(x,s)}{\partial s}dx + f(a_2(s),s)\frac{\partial a_2(s)}{\partial s} - f(a_1(s),s)\frac{\partial a_1(s)}{\partial s}$$

- Therefore,

$$\frac{\partial EC_U(s,Q)}{\partial s} = h - \frac{bE[D]}{Q}\int_s^{\infty} f_{D_L}(x_L)dx_L = 0$$

$$\Rightarrow \int_{s^*}^{\infty} f_{D_L}(x_L)dx_L = \frac{hQ^*}{bE[D]} \Rightarrow 1 - F_{D_L}(s^*) = \frac{hQ^*}{bE[D]}$$

- The two equations giving Q^* and s^*, generally have no closed-form solution except for some lead time demand distribution (e.g., uniform, exponential).
- In the general case where no closed-form solution exist, the optimal values Q^* and s^* can be found numerically.

- **Example 7.**
  - Electro uses resin in its manufacturing process at the rate of 1000 gallons per month. Inventory for the resin is reviewed continuously. It costs Electro \$100 to place an order for a new shipment. The holding cost is \$2 per gallon per month, and the shortage cost per gallon is \$10. Historical data show that the demand during lead time is uniform over the range (0, 100) gallons.
  - Determine the optimal ordering policy for Electro.
  - In this case, $E[D] = 1{,}000$, $D_L \sim U(0, 100)$, $K = 100$, $h = 2$, $b = 10$.
  - In this case,

$$f_{D_L}(x_L) = \begin{cases} 1/100, & 0 \le x_L \le 100 \\ 0, & \text{otherwis} \end{cases}$$

$$F_{D_L}(x_L) = \begin{cases} x_L/100, & 0 \le x_L \le 100 \\ 1, & x_L > 100 \end{cases}$$

  - Then, the optimality equations can be solved as follows.

$$1 - \frac{s^*}{100} = \frac{hQ^*}{bE[D]} \Rightarrow s^* = 100\left(1 - \frac{hQ^*}{bE[D]}\right) = 100 - 0.02Q^*$$

$$Q^* = \sqrt{\frac{2E[D](K + bS)}{h}}$$

where

$$S = \int_{s^*}^{100} [(s^* - x_L)/100] dx_L = \left.\frac{(s^* - x_L)^2}{200}\right|_{s^*}^{100} = \frac{(s^* - 100)^2}{200}$$

$$= \frac{(0.02Q^*)^2}{200} = 0.000002Q^{*2}$$

$$\Rightarrow Q^* = \sqrt{\frac{2000\left(100 + 10(0.000002Q^{*2})\right)}{2}} = \sqrt{100,000 + 0.02Q^{*2}}$$

$$\Rightarrow Q^{*2} = 100,000 + 0.02Q^{*2}$$

$$\Rightarrow Q^* = \sqrt{100,000/0.98} \approx 319$$

$$\Rightarrow s^* = 100 - 0.02 \times 317 \approx 94$$

- The optimal policy in this case is as follows:

  When inventory drops below 94, place an order for 319.

- **Example 8.**
  - A large military installation stocks a special purpose vacuum tube for use in radar sets under continuous review. The average annual demand for this tube is 1,600 units. Each tube costs $50. The cost of placing an order for the tube is $4,000. Inventory holding cost is estimated to be 10 $/unit/year. It has been found that if a demand occurs when the system is out of stock, it is possible to obtain the tube from another location at an expense of $2,000 over the cost of the unit. An empirical investigation has shown that that the distribution of lead time demand is normal with mean 750 and standard deviation 50.

- Determine the optimal ordering policy for the tube.
- Since lead time demand is normal, then the optimality equations for s^* and Q^* cannot be solved directly as in the uniform demand case in Example 7.
- Here, we'll evaluate the expected annual cost for given values of s and Q, and then we'll find s and Q numerically using Excel solver.
- When the lead time demand is normal with mean μ_L and standard deviation σ_L, it can be shown that

$$\int_s^{\infty}(s-x_L)f_{D_L}(x_L)dx_L=(\mu_L-s)\left[1-\Phi\left(\frac{s_L-\mu_L}{\sigma}\right)\right]+\sigma_L\phi(\frac{s_L-\mu_L}{\sigma})$$

where ϕ and Φ are the pdf and cdf of the standard normal random variable.

- Then, the expected annual cost is

$$EC_U(s,Q)\approx\frac{KE[D]}{Q}+cE[D]+h\left(\frac{Q}{2}+s-E[D_L]\right)$$
$$+\frac{bE[D]}{Q}\left\{(\mu_L-s)\left[1-\Phi\left(\frac{s-\mu_L}{\sigma_L}\right)\right]+\sigma_L\phi(\frac{s-\mu_L}{\sigma_L})\right\}.$$

- Then, using Excel solver, we find $s^* = 884$, $Q^* = 1{,}147$, and the optimal annual cost is \$92,813. (See Excel file.)
- The optimal policy in this case is as follows:

When inventory drops below 884*, place an order for* 1147.

- **The single-period newsvendor model**
  - Consider a newsvendor, who at a start of each day, must decide the amount of newspapers to stock, S.
  - Placing an order has a negligible cost.
  - Daily demand for the newspaper is D (a random variable).
  - If demand during the day is less than S, than a holding cost at a rate h per unit is charged for each unit remaining in inventory.
  - If demand during the days is greater than S, than a shortage cost b is charged on each unit remaining in inventory.
  - By conditioning on daily demand, the expected daily cost is

$$EC(S) = h\int_0^S (S-x) f_D(x)dx + b\int_S^\infty (x-S) f_D(x)dx$$

    where $f_D(.)$ is the pdf of D. (Denote by $F_D(.)$ the cdf of D.)
  - Differentiating with respect to S (using Leibniz rule) gives

$$\frac{\partial EC(S)}{\partial S} = h\int_0^S f_D(x)dx - b\int_S^\infty f_D(x)dx = hF_D(S) - b(1-F_D(S))$$

$$\frac{\partial^2 EC(S)}{\partial S^2} = (h+b) f_D(S) > 0$$

  - It follows that $EC(S)$ is convex in S with an optimal order quantity given by

$$F_D(S^*) = P\{D < S^*\} = \frac{b}{b+h}.$$

- **Newsvendor model facts**
  - The newsvendor has many applications beyond the newspaper case. E.g., it applies to perishable goods (e.g., produce, bread, etc.) and to style clothing.
  - The model can be even applied in manufacturing when deciding on how much to produce in a single batch.
  - The ratio $b/(b+h)$ is known as the *critical fractile*. This ratio can be written as $c_u/(c_u+c_o)$, where c_u and c_o are the unit cost of underage and overage.
  - There are different derivations of the newsvendor model that consider other parameters such as unit variable cost c, unit salvage value, v, and selling price r.
  - These parameters can be included into the model easily. E.g., setting $h' = h + c - v$ and $b' = b + (r - c)$, allows deriving S^* via a critical fractile $b' / (b' + h')$.
  - If the period starts with an initial on hand, inventory, x, then the optimal policy is to order $S^* - x$ if $x < S^*$ and not order otherwise, where S^* is as given above. (This is called an *order-up-to* policy.)
  - This follows because $EC(S)$ is convex in S.
  - If the initial inventory is $x < S^*$, then the optimal order quantity is $Q^* = S^* - x$, which minimizes expected cost.
  - If $x \geq S^*$, then any order quantity $Q > 0$ increases cost.

- **Example 9.**
  - The owner of a newsstand wants to determine the number of *USA now* newspapers that must be ordered at the beginning of each day. The owner pays ¢30 per copy and sells it for ¢75. Newspapers left at the end of the day are sold for recycling purposes at a price of ¢5. Daily demand is assumed to be normally distributed with mean 300 and standard deviation 20.
  - In this case, the order quantity can be found by defining equivalent holding and penalty costs, $h' = 30 - 5 =$ ¢25 and $p' = 75 - 30 =$ ¢45.
  - Let μ and σ be the mean and standard deviation of daily demand. The optimal order quantity is given by

$$P\{D < S^*\} = \frac{b'}{b'+h'} \Rightarrow P\{Z < \frac{S^*-\mu}{\sigma}\} = \frac{b'}{b'+h'}$$

$$\Rightarrow S^* = \mu + \sigma\Phi^{-1}\left(\frac{b'}{b'+h'}\right),$$

where $\Phi^{-1}(.)$ is the inverse of the standard normal cdf.

  - Therefore, the optimal order quantity is 307, derived as

$$S^* = 300 + 20\Phi^{-1}\left(\frac{45}{45+30}\right) = 300 + 20\Phi^{-1}(0.643)$$

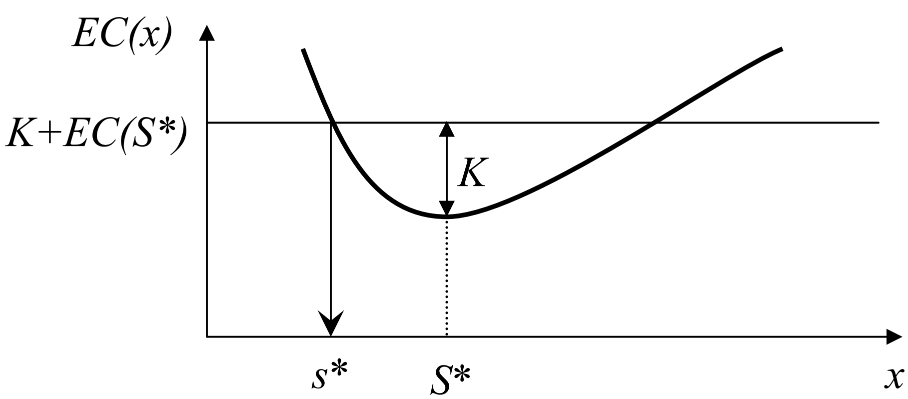
$$\approx 300 + 20 \times 0.37 \approx 307$$

  - Note that $\Phi^{-1}(0.643) \approx 0.37$ was found from Table B.1.

- **Single-period model with setup cost**
  - Suppose that in the single-period model there is a fixed cost K for placing an order, and that the periods starts with an initial stock x.
  - In some situations, even if x is below the optimal order-up-to-level, S^*, it may not be efficient to order because of the fixed cost.
  - Specifically, one should order if the expected cost associated with placing an order is higher than that of associated with not placing an order.
  - First, if $x \geq S^*$, then no order should be placed as above.
  - Now, if $x < S^*$, then the optimal-order-up-to level is S^*.
  - Therefore, the cost of ordering is $K + EC(S^*)$.
  - The cost of not ordering is obviously $EC(x)$.
  - Then, an order should be placed if

$$K + EC(S^*) < EC(x) \ .$$

  - The convexity of $EC(x)$ implies that there exists a unique s^*, such that $K + EC(S^*) < ECO(x)$, for $x < s^*$.



- The optimal *reorder point* s^* can be found by solving

$$K + EC(S^*) = EC(s^*) .$$

- Note that s^* solving this equation may be negative. In this case, s^* is set to zero and the optimal policy is not to order.
- This could be the case if the fixed cost K is too high (see Example 14.2-2, text).
- The optimal (s, S) policy is then

If $x < s^*$, order $S^* - x$. Otherwise, do not order.

- **Example 10.**
  - The daily demand for an item is uniformly distributed between 0 and 10 units. The unit holding cost of the item during the period is \$0.5/unit and the unit shortage cost is \$4.50/unit. A fixed cost of \$5 is incurred every time an order is place.
  - Determine the optimal inventory policy for the item.
  - In this case, the demand pdf and cdf are

$$f_D(x) = \begin{cases} 1/10, & 0 \le x \le 10 \\ 0, & \text{otherwis} \end{cases} \qquad F_D(x) = \begin{cases} x/10, & 0 \le x \le 10 \\ 1, & x>10 \end{cases}$$

  - The optimal order-up-to level is S^* such that

$$F_D(S^*) = \frac{b}{b+h} \Rightarrow \frac{S^*}{10} = \frac{4.5}{4.5+0.5} \Rightarrow S^* = 9 .$$

  - To determine the reorder point s^*, we need to write an explicit expression for $EC(S)$.

$$EC(S) = h\int_0^S (S-x) f_D(x)dx + b\int_S^{\infty} (x-S) f_D(x)dx$$
$$= 0.5\int_0^S (S-x)(1/10)dx + 4.5\int_S^{10} (x-S)(1/10)dx$$
$$= -0.05(S-x)^2/2\Big|_0^S + 4.5(x-S)^2/2\Big|_S^{10}$$
$$= 0.025S^2 + 0.225(10-S)^2$$
$$= 0.25S^2 + 4.5S + 22.5$$

- Then, s^* is the solution to the following equation

$$5 + (0.25\times 10^2 - 4.5\times 10 + 22.5) = 0.25s^{*2} - 4.5s^* + 22.5$$
$$\Rightarrow 0.25s^{*2} - 4.5s^* + 15 = 0 \Rightarrow s^* = 4.417 \Rightarrow s^* \approx 5$$

- The optimal policy is

  Order up to 9 (i.e. order 9 − x), if initial inventory x < 5.

  Otherwise, do not order.

- **Multi-period periodic review models**
  - The single period model can be generalized into multi-period model with n periods, where inventory is reviewed at the beginning of every period, and inventory is carried over when period to the other.
  - Suppose that excess demand is backordered.
  - The objective of the analysis is to decide how much to order in each period t, t =1, …, n, given that the initial inventory in period t is x_t .

- Suppose that demands in periods 1 to *n* are iid.
- Then, similar to the time-varying demand case, the problem can be solved with dynamic programming, with the objective of minimizing the cost-to-go between periods *t* and *n*.
- However, the explicit solution to this *stochastic dynamic program* is complex.
- Still we can tell the form of the optimal policy in these cases.
- If the ordering cost is zero in all periods, then the ordering policy is an order-up-to level in all periods.
- That is, there exist critical values, S_1*, S_2*, …, S_n*, such that it is optimal to order in period *t* if $x_t < S_t$* .
- If the setup cost is not zero, then the optimal policy is of the (s_t, S_t) type in all periods.
- Note that these results hold if there is a deliver lead time. That is, if orders places at the beginning of period *t* are delivered at the beginning of period $t+L$.
- However, with a delivery lead time the ordering decision is based on reviewing the *inventory position* (see below).
- Interestingly, for the infinite horizon case, $n = \infty$, the optimal policy is stationary (i.e., the critical numbers are equal in all periods) as discussed next.

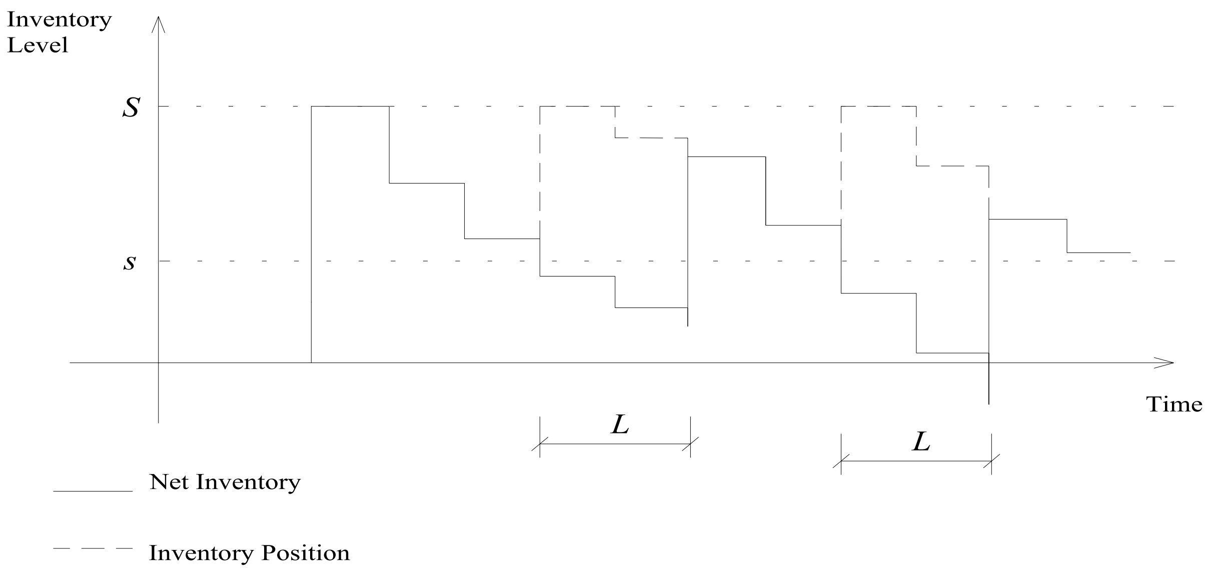
- **Infinite horizon periodic review models – zero lead time**
  - Consider the model discussed above with an infinite number of periods.
  - Let D be the demand per period.
  - Suppose the holding cost is h per each unit leftover in inventory at the end of a period, and every unit of backlogged demand incurs a cost b.
  - If there is no fixed cost for placing an order, then it can be shown that the optimal policy is of an order-up-to-level with a critical number S^*, which is the same for all periods.
  - However, because of the lead time, the decision to order is based on reviewing the *inventory position*, defined as the inventory on hand plus on order minus backorder.
  - That is, if the inventory position at the beginning of a period is less than S^* than an order is placed to bring the inventory position up to S^*.
  - The order-up-to-level S^* is given by a critical-fractile formula as

$$F_{D^{L+1}}(S^*) = P\{D^{L+1} < S^*\} = \frac{b}{b+h},$$

where D^{L+1} is the $(L+1)^{\text{th}}$ fold convolution of the demand.

  - That is $D^{L+1} = \sum_{i=1}^{L+1} D_i$, where the D_i's are iid random variables representing the demand in L+1 periods.

- For example, if the demand per period is normal with mean μ and standard deviation σ, i.e., $D \sim N(\mu, \sigma)$. Then, $D^{L+1} \sim N[(L+1)\mu, \sqrt{L+1}\sigma]$.
- When a fixed setup cost K is incurred every time an order is placed, then the optimal policy is of the (s, S) type *based also on inventory position*.



- However, there is no simple exact formula for determining the optimal (s, S) values.
- Several approximations have been developed for determining near-optimal (s, S) values.
- The *revised power approximation method* determines (s, S) values which have been shown to be very close to optimal.

- This method which requires only knowing the mean and the variance of the demand per review period, μ and σ, works as follows.

Step 1. Find the mean and standard deviation of demand in L+1 periods, $\mu_{L+1} = \mu(L+1)$, and $\sigma_{L+1} = \sigma(L+1)^{0.5}$.

Step 2. Find the following.

$$Q_p = 1.3\mu^{0.494}\left(\frac{K}{h}\right)^{0.506}\left(1+(\sigma_{L+1}/\mu)^2\right)^{0.116}$$

$$z = \left(\frac{Q_p}{\sigma_{L+1}(p/h)}\right)^{0.5},$$

$$s_p = 0.973\mu_{L+1} + \sigma_{L+1}\left(\frac{0.183}{z} + 1.063 - 2.192z\right).$$

Step 3. If $Q_p/\mu > 1.5$, then the "optimal" policy is $s^* = s_p$ and $S^* = s_p + Q_p$, stop. Otherwise, go to Step 4.

Step 4. The optimal policy is $s^* = \min(s_p, S_0)$ and $S^* = \min(s_p + Q_p, S_0)$, where $S_0 = \mu_{L+1} + \sigma_{L+1}\Phi^{-1}\left(\frac{p}{p+h}\right)$ with $\Phi^{-1}(.)$ being the inverse cumulative density function of the standard normal random variable.

Remark.

The optimal policy for the case with zero lead time can be obtained by setting $L = 0$ in both cases with and without fixed cost.

- **Example 11.**
  - An item inventory is reviewed every half month. Demand during this review period is normal with mean 50 and standard deviation 20. The fixed cost for placing an order is negligible. An order is delivered after a lead time of 1 month. Unfilled demand is backlogged. The holding and shortage costs (based on end of period inventory) are 0.02 \$/unit and 0.2 \$/unit respectively.
  - Determine the optimal order quantity for this item.
  - Since the fixed cost is zero, an order-up-to policy is optimal. The optimal order up to level, S^*, is determined as follows.

$$F_{D^{L+1}}(S^*) = \frac{b}{b+h},$$

where $L = 2$, $D^{L+1} \sim N(150, 20\sqrt{3})$, $b = 0.02$ and h = 0.2.

  - Then,

$$S^* = \mu_{L+1} + \Phi^{-1}[b/(b+h)]\sigma_{L+1} = 150 + \Phi^{-1}(0.2/0.22) \times 36.64$$
$$\approx 150 + 1.34 \times 36.64 \approx 199$$

➢ The optimal policy is

At the beginning of a period, if the inventory position is less than 199, place an order that brings the inventory position up to 199 units. Otherwise, do not order.

- **Example 12.**
  - ➢ Redo Example 11 assuming that there is a fixed ordering cost of \$25.
  - ➢ Now the optimal policy is of the (s, S) form. We use the revised power approximation method to get a good policy.

Step 1 $\mu_{L+1} = \mu(L+1) = 50 \times 3 = 150$

$$\sigma_{L+1} = \sigma(L+1)^{0.5} = 5\sqrt{3} = 36.64$$

Step 2.

$$Q_p = 1.3\mu^{0.494}\left(\frac{K}{h}\right)^{0.506}\left(1+(\sigma_{L+1}/\mu)^2\right)^{0.116}$$

$$= 1.3(50)^{0.494}\left(\frac{25}{0.02}\right)^{0.506}\left(1+(36.64/50)^2\right)^{0.116} = 348.28$$

$$z = \left(\frac{Q_p}{\sigma_{L+1}(p/h)}\right)^{0.5} = \left(\frac{348.28}{36.64(0.2/0.02)}\right)^{0.5} = 0.975$$

$$s_p = 0.973\mu_{L+1} + \sigma_{L+1}\left(\frac{0.183}{z} + 1.063 - 2.192z\right)$$

$$= 0.973 \times 150 + 36.64\left(\frac{0.183}{0.975} + 1.063 - 2.192 \times 0.975\right) \approx 108$$

Step 3. $Q_p / \mu = 348.28/50 > 1.5$ then the "optimal" policy is $s^* = 108$ and $S^* = 108 + 348.28 \approx 456$,

- The approximately optimal policy is
  At the beginning of a period, if the inventory position is less than 108, place an order that brings the inventory position up to 456 units. Otherwise, do not order.