Inventory Theory (1)

- **Introduction**
  - Most industries have to deal with inventories. E.g., shelf and warehouse (back room) inventory in retail, and raw material, work in process and finished product inventory in manufacturing.
  - Even services that do not keep physical inventory can treat their resources as perishable inventory. E.g., seats on a plane, rooms in a hotel.
  - Proper inventory management is crucial for the success of any business.
  - E.g., holding too much inventory involves "tying" large capitals, while holding little inventory leads to lost revenue and dissatisfied customers.
  - To get an idea of the importance of inventory management, note that the capital invested in U.S. inventories was estimated to be $1 trillion in 1991.
  - In an inventory system, inventory is depleted by customer demand and is replenished from orders to suppliers.
  - The key questions in inventory management are
    - When to order?
    - How much to order?

- **Why hold inventory?**
  - To benefit from *economies of scale*.
  - To cope with *demand variability over time*.
  - To protect against *demand uncertainty*.
  - To protect against *supply uncertainty*.

- **Inventory costs**
  - The two key questions of when and how much to order are commonly answered based on a cost minimization criteria.
  - There are three types of costs in an inventory system:
    - The cost to place an order.
    - The cost of holding inventory.
    - The penalty cost for unmet demand.
  - The *order cost*, $OC(x)$, is a function of the order size, x, and is generally given as

$$OC(x) = \begin{cases} 0 & \text{if } x = 0 \\ K + cx & \text{if } x > 0 \end{cases}$$

  - Here $K > 0$, is the *fixed ordering cost* which is charged every time an order is placed regardless of its size.
  - Costs included in K typically include book-keeping costs associated with processing an order, fixed transportation costs, and order handling costs.
  - And c is *unit variable cost*, assumed (for now) independent of x. We'll see that variable cost can also depend on x.

- The *holding cost* or the *inventory carrying cost* is the cost associated with storage of inventory until it is sold or used.
- It includes the cost of storage space, insurance, taxes, and most importantly, *the cost of capital* tied up in inventory.
- Think of the cost of capital as the interest paid to a bank that finances purchasing from suppliers.
- The holding cost is computed function of the amount of inventory on hand.
- It is assessed based on the average on-hand inventory or based on inventory level at the end of a certain period.
- The *penalty cost*, also known as *shortage cost*, is the cost of not being able to meet customer demand when it occurs.
- The penalty cost depends on whether the excess demand is *backordered* (also known as *backlogged*) or *lost*.
- In a backorder demand situation, a customer not finding any inventory on hand is willing to wait until an order arrives to receive his request.
- In a lost demand situation, a customer not finding any inventory on hand, will not wait (and perhaps look for his request elsewhere).
- In reality, there can be a combination of backorder and lost sales.

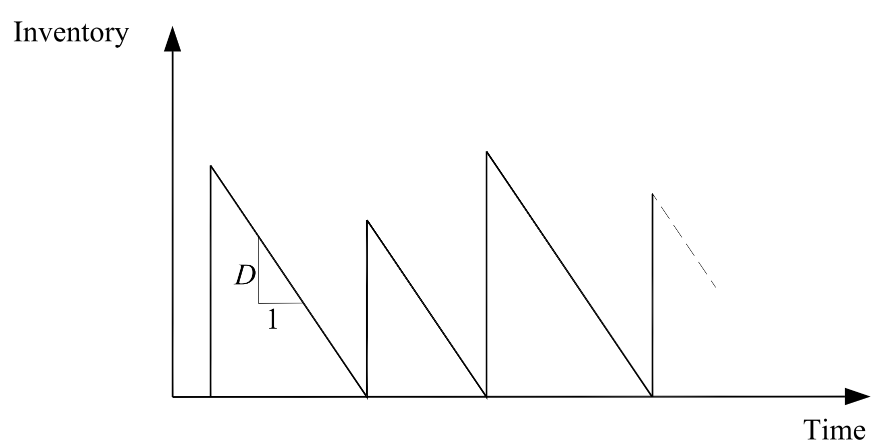
- **Estimating Inventory costs**
  - The ordering cost is usually easy to estimate.
  - Holding cost is somewhat easy to estimate based on the firm cost of capital, which is found as a weighted average of the interest rates from different sources of funding.
  - However, estimating the holding cost can be complicated in some situations (e.g. when payment to suppliers is delayed).
  - The penalty cost is the most difficult to estimate. How do you measure "loss of customer goodwill"?
  - Because difficulty in its estimation, sometimes a *service level constraint* is used instead of a penalty cost.

- **Characteristics of inventory systems**
  - When constructing an inventory model we should specify assumptions in the following 10 areas.
    1. *Demand*: Can be (i) deterministic (known with certainty) and constant over time (stationary), (ii) deterministic and time varying (dynamic), (iii) probabilistic and stationary, or (iv) probabilistic and dynamic.
    2. *Lead time*: This is the time until an order arrives. It can be (i) zero (i.e. instantaneous delivery), (ii) non-zero and deterministic, or (iii) random.

3. *Review time*: This defines how often inventory is reviewed. There are two basic variations. Under *continuous review*, the inventory is monitored continuously. Under *periodic review*, the inventory is checked periodically (e.g. once per day, per week, etc.)
4. *Excess demand*: Could be (i) backordered or (ii) lost.
5. *Planning horizon*: This specifies the time interval the model considers. It can be (i) a single period, (ii) a finite number of periods, or (iii) an infinite horizon.
6. *Other factors that consume inventory*: E.g., perishable inventory (e.g. in case of food inventory), obsolescence (e.g. clothing and electronics inventory), etc.
7. *Supply*: Supply is often assumed certain (i.e. the amount received is equal to the amount ordered). However, there are *random supply* situations (e.g. due to quality problems and human errors).
8. *Location*: Classic inventory models consider inventory at a single location. Other (recent) *supply chain management*-type models consider inventory in many locations (e.g., a warehouse and multiple retail outlets).
9. *Number of items*: Most inventory models consider a single item. Other models consider multiple items competing over demand or space.

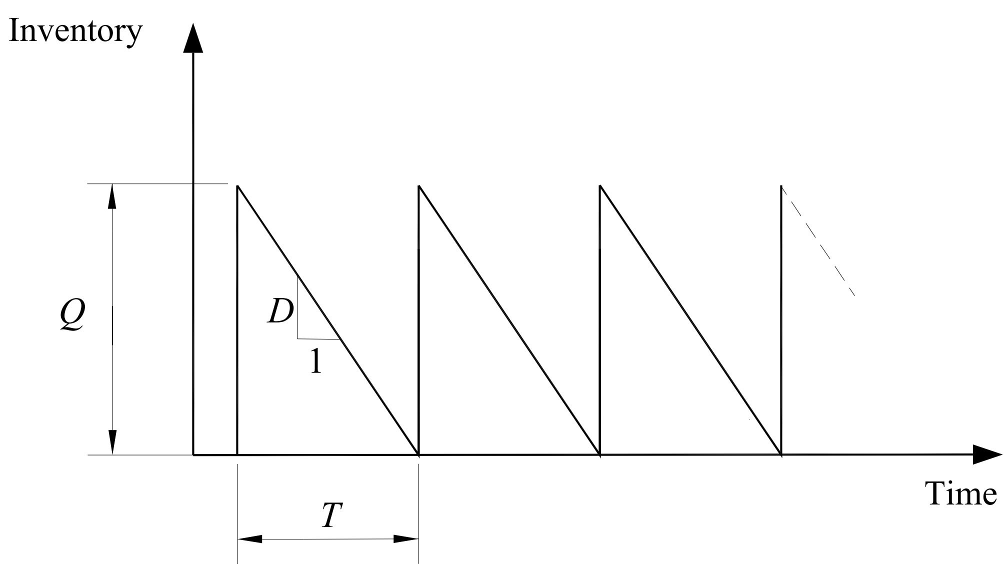
10. *Centralized/decentralized decision making*: This defines whether different products and locations are managed by one or more decision makers in a centralized manner.

- **The classic Economic Order Quantity (EOQ) model**
  - Consider a facility (e.g., a retailer or a warehouse) that faces a constant demand for a single item at a rate D per unit time.
  - Placing an order of size $Q > 0$ to a supplier costs $K+cQ$ dollars, where K and c are the fixed and variable order costs.
  - Holding inventory incurs a unit cost h (\$/unit/unit time) which is proportional to average inventory level.
  - The lead time until an order is received is zero.
  - No shortages are allowed.
  - The planning horizon is infinite.
  - The objective is to determiner the ordering policy that minimizes the cost per unit time (e.g. annual cost).
  - When to order in such a situation?
  - When the inventory level reaches 0 (saves on holding cost).



- In terms of how much to order, it can be shown that it is optimal to order in equal quantities.
- Let Q be the order size.
- The EOQ policy can be summarized as follows:

  Whenever the inventory level reaches zero, order Q.
- Now we can determine the optimal value of Q that minimizes cost per unit time.
- First, we find the cost per ordering cycle with duration $T = Q/D$.



- The cost per ordering cycle is the sum of holding and ordering cost.
- Let $I(t)$ be the inventory level at time t. The holding cost per cycle is

$$\int_0^T hI(t)dt = h \times (\text{area under inventory level}) = hQ^2/(2D).$$

- The ordering cost per cycle is $K + cQ$.
- Then, the cost per ordering cycle is

$$C_T(Q) = K + cQ + hQ^2/(2D).$$

- The cost per unit time is

$$C_U(Q) = \frac{C_T(Q)}{T} = \frac{K + cQ + hQ^2/(2D)}{Q/D} = K\frac{D}{Q} + cD + h\frac{Q}{2}.$$

- Differentiating implies that

$$\frac{\partial C_U(Q)}{\partial Q} = -\frac{KD}{Q^2} + \frac{h}{2},$$

$$\frac{\partial^2 C_U(Q)}{\partial Q^2} = \frac{2KD}{Q^3} > 0.$$

- Therefore, $C_U(Q)$ is convex, and the optimal order quantity that minimizes $C_U(Q)$, Q^*, is found by setting the first derivative equal to zero as

$$Q^* = \sqrt{\frac{2KD}{h}}.$$

- The corresponding optimal (minimum cost), $C_U(Q^*)$ is

$$C_U^* = cD + \sqrt{2KDh}.$$

- **EOQ facts**
  - Note that Q^* is independent of the variable cost c. So some text books do not include the variable cost in the derivation.
  - The holding cost per unit time could have been derived directly as $h\times$(average inventory level) $= h\times Q/2$.

- The ordering cost per unit time could also be found directly as $(K + cQ)\times$(# of orders per unit time) $= (K + cQ)(D/Q)$.
- The optimal order quantity, Q^*, is increasing in the ordering cost K. That is, high ordering cost implies ordering less frequently in large quantities.
- In addition, Q^*, is decreasing in the holding cost h. That is, high holding cost implies ordering more frequently in small quantities.
- In fact, Q^* explicitly balances ordering and holding cost, since for $Q = Q^*$ the holding and fixed ordering costs per unit time are equal: $hQ^*/2 = KD/Q^*$.
- If an order requires a lead time $L > 0$ to arrive. Then, the optimal policy is to order Q^*, L time units before the inventory level reaches zero.
- Specifically, the *reorder inventory level* is DL if $L < T^* = Q^*/D$.
- If $L > T^*$, the reorder level is $D\times$(fractional part of L/T^*).
- An important property of EOQ is that the optimal cost, $C_U(Q)$, is not too sensitive to Q when Q is close to Q^*.
- Specifically, a relative deviation of q from Q^* by using an order quantity $Q' = (1+q)Q^*$ leads to a relative change of the cost per unit time (excluding variable cost) of $q^2/ [2(1+q)]$.

- For example a deviation of 20% in Q^* increase cost by $0.2^2/[2(1+0.2)] = 0.017$, by less than 2%.
- This is an important feature in practice when accurate estimates of parameters are not available or when the exact optimal order quantity cannot be used.

- **Example 1.**
  - Consider a three-ohm resistor used in the assembly of an automated processor of X-ray films. The demand for this item has been relatively level over time at a rate of 2,400 units/year. The unit holding cost of the resistor is 0.096 \$/unit/year, and the fixed ordering cost is \$3.2.
  - What is the optimal order quantity and corresponding annual cost? (Ignore variable cost).
  - The optimal order quantity is

$$Q^* = \sqrt{\frac{2KD}{h}} = \sqrt{\frac{2 \times \$3.2 \times 2400 \text{ units/year}}{\$0.096 \text{ unit/year}}} = 400 \text{ units.}$$

$$C_U{}^* = \sqrt{2KDh} = \sqrt{2 \times \$3.2 \times 2400 \text{ units/yr} \times \$0.096 \text{ unit/year}}$$
$$= \$38.4 \,/\, \text{year}$$

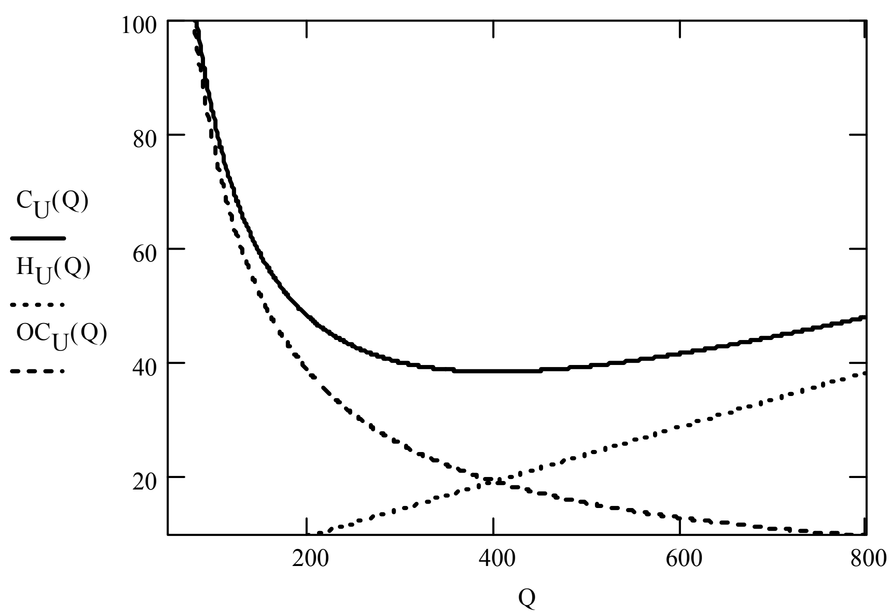
  - Note that the optimal order quantity involves ordering 2-month supply of the resistor every two months (since $T^* = Q^*/D = 2/12$ years = 2 months).
  - Note also that the optimal order cost $KD/Q^* = \$19.2$ is equal to the optimal holding $hQ^*/2 = \$19.2$.

- Suppose that it takes a lead time of one week to receive an order. How should the resistor be ordered?
- The optimal policy is to order 400 units when the inventory level reaches $DL = 2400/52 \approx 46$ units.
- Suppose the supplier will not deliver below a three-month supply of the transistor? How will this affect the cost?
- This involves using a $Q' = 3/2\, Q^*$ instead of Q^*, which will increase the annual cost by $0.5^2/[2(1+0.5)] = 0.083 \approx 8\%$.
- This can be verified by evaluating $C_U(Q) = K\dfrac{D}{Q} + h\dfrac{Q}{2}$,

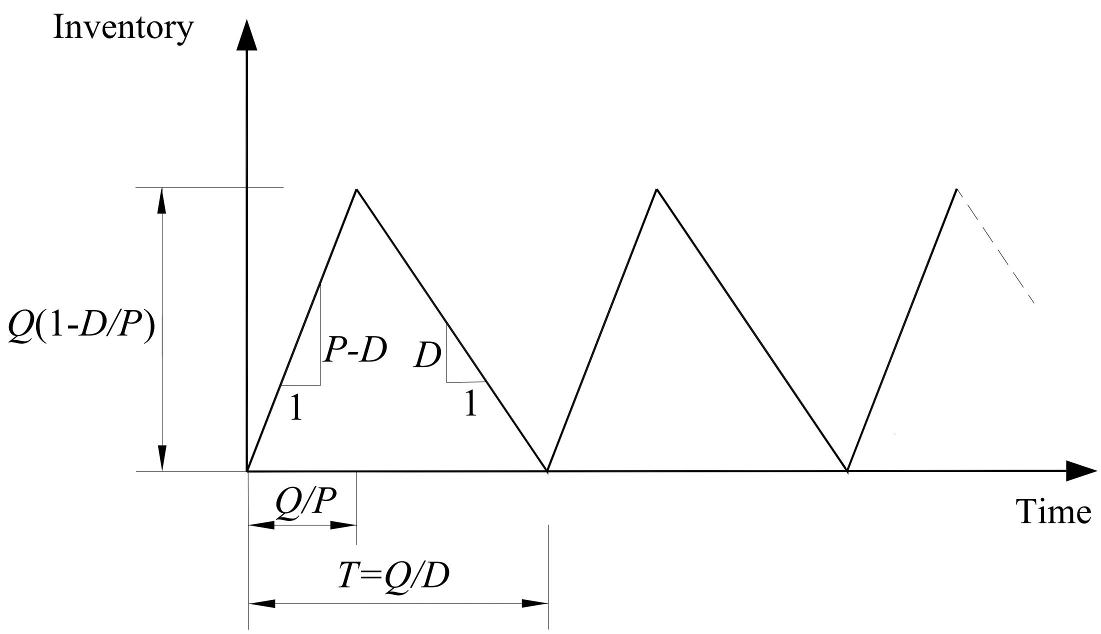
  for $Q = 600$, which gives $C_U(600) = \$41.6$, which is

  $$(41.6 - 38.4)/38.4 = 0.083 = 8.3\%$$

  above the optimal cost.
- Finally, it is instructive to plot the annual ordering, holding, and total cost on the same graph.



- **EOQ model with a finite production rate (EPQ model)**
  - In certain systems, especially production systems, the whole order quantity is not delivered at the same time.
  - The batch is instead delivered continuously according to a certain production rate P per unit time.
  - The economic production quantity model (EPQ) is the EOQ model but with orders delivered according to a rate $P > D$.
  - Adopting an order-at-zero-inventory policy order quantities all equal to Q, the inventory over time varies as follows.



  - The time to receive an order is Q/P. During this time the inventory level rises from zero to $Q - D(Q/P) = Q(1-D/P)$.
  - Then, it can be shown the average inventory level in an ordering cycle is $[Q(1-D/P)\times T/2]/T = Q(1-D/P)/2$.

- Note that cycle duration continues to be $T = Q/D$.
- Therefore, the expected cost per unit time can be written as

$$C_U(Q) = K\frac{D}{Q} + cD + h\frac{Q(1-D/P)}{2}.$$

- The optimal order quantity and cost are derived similar to the EOQ model case as

$$Q^* = \sqrt{\frac{2KD}{h(1-D/P)}}, \quad C_U{}^* = \sqrt{2KDh(1-D/P)}\ .$$

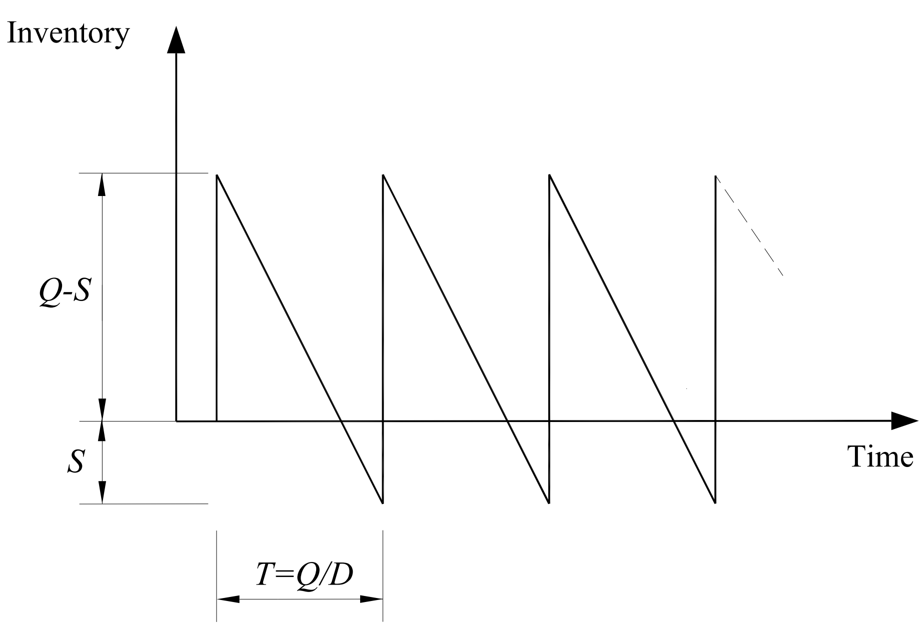
- The optimal order quantity for the EPQ model is less than that of the EOQ model because inventory spends less time on hand in the EPQ model, which reduces holding cost.
- As $P \to \infty$, Q^* converges to the EOQ order quantity.

- **Example 2.**
  - Suppose that the resistor in Example 1 is replenished according to a production rate of 900 per month. What are the optimal order quantity and annual cost?
  - In this case, P = 800 units/month = 9,600/year. Then, D/P = 1/4, and

$$Q^* = \sqrt{\frac{2KD}{h(1-D/P)}} = \sqrt{\frac{2\times 3.2\times 2400}{0.096(1-1/4)}} \approx 462 \text{ units.}$$

$$C_U{}^* = \sqrt{2KDh(1-D/P)} = \sqrt{2\times 3.2\times 2400\times 0.096(1-1/4)}$$
$$= \$33.3/\text{year}$$

- **EOQ model with shortages allowed**
  - Suppose now that in the EOQ model shortages are allowed and backordered.
  - Suppose that the unit backorder cost is π \$/unit/unit time. T his can be seen as a waiting cost.
  - The idea here that we may be able to save on order costs by making some customers wait and ordering less frequently.
  - The shortage cost per unit time is proportional to the average backorder level (similar to the holding cost).
  - With a policy that orders in equal amounts, of size Q, the problem is to determine the optimal value of Q and of the backorder level S at the end of an ordering cycle.
  - The inventory varies as follows over time.



- The average inventory level is

$$\{(Q-S)\times[(Q-S)/D]/2\}/T=(Q-S)^2/2Q\ .$$

- The average backorder level is

$$\{S\times[S/D]/2\}/T=S^2/2Q\ .$$

- Therefore, the cost per unit time is

$$C_U(Q,S)=K\frac{D}{Q}+cD+h\frac{(Q-S)^2}{2Q}+\pi\frac{S^2}{2Q}.$$

- It can be easily shown that $C_U(Q,S)$ is convex in S (for a fixed value of Q) , since

$$\frac{\partial C_U(S,Q)}{\partial S}=-\frac{h(Q-S)}{Q}+\frac{\pi S}{Q},$$

$$\frac{\partial^2 C_U(Q)}{\partial S^2}=\frac{h+\pi}{Q}>0\,.$$

- Then, the optimal value of S that minimizes cost for Q fixed, is obtained by setting the first derivative equal to zero as

$$S^*=Q\frac{h}{\pi+h}.$$

- Now the cost "at optimal S" is

$$\begin{aligned}C_U(Q)&=K\frac{D}{Q}+cD+h\frac{Q\{1-[h/(\pi+h)]\}^2}{2}+\pi\frac{Q[h/(\pi+h)]^2}{2}\\&=K\frac{D}{Q}+cD+h\frac{Q[\pi/(\pi+h)]^2}{2}+\pi\frac{Q[h/(\pi+h)]^2}{2}\\&=K\frac{D}{Q}+cD+\frac{Q}{2}\frac{\pi h}{\pi+h}\end{aligned}$$

- Therefore, the optimal order quantity is found similar to the EOQ model as

$$Q^* = \sqrt{\frac{2KD}{\pi h/(\pi + h)}} = \sqrt{\frac{2KD}{h}}\sqrt{\frac{\pi + h}{\pi}}\ .$$

- In addition, the optimal backorder level is

$$S^* = Q^*\frac{h}{\pi + h} = \sqrt{\frac{2KD}{\pi}}\sqrt{\frac{h}{\pi + h}}\ .$$

- The optimal cost is $C_U{}^* = cD + [\pi/(\pi + h)]\sqrt{2KDh}$.
- Note that the optimal order quantity increases relative to the EOQ model order quantity as a result of backorders.
- Note also that as $\pi \to \infty$, Q^* converges to the EOQ order quantity.

- **Example 3.**
  - Suppose that demand resistor in Example 1 can be backordered at a cost of \$0.5/unit/year. What are the optimal order quantity backorder level and annual cost?
  - Here $\pi = 0.5$ and $\pi + h = 0.596$. Then,

$$Q^* = \sqrt{\frac{2KD}{h}}\sqrt{\frac{\pi + h}{\pi}} = 400\sqrt{\frac{0.596}{0.5}} \approx 437 \text{ units}$$

$$S^* = \sqrt{\frac{2KD}{\pi}}\sqrt{\frac{h}{\pi + h}} = \sqrt{\frac{2 \times 3.2 \times 2400}{0.5}}\sqrt{\frac{0.596}{0.5}} \approx\ 161 \text{ units}$$

$$C_U{}^* = [\pi/(\pi + h)]\sqrt{2KDh} = (0.5/0.596) \times 38.4 = \$32.2\ .$$

- **EOQ model with quantity discounts**
  - Suppose that in the EOQ model the supplier offers a price discount for large quantities.
  - Specifically, suppose the unit variable order cost is

$$c(Q) = \begin{cases} c_1, & \text{if } Q < \bar{Q} \\ c_2, & \text{if } Q \geq \bar{Q} \end{cases},$$

    where $c_2 < c_1$.
  - In addition, since a major part of the holding cost is related to cost of capital (which is proportional to variable cost) the holding cost will be of the form[1]

$$h = \begin{cases} h_1, & \text{if } Q < \bar{Q} \\ h_2, & \text{if } Q \geq \bar{Q} \end{cases},$$

    where $h_2 < h_1$.
  - The cost per unit time is then given by

$$C_U(Q) = \begin{cases} C_U^1(Q), & \text{if } Q < \bar{Q} \\ C_U^2(Q), & \text{if } Q \geq \bar{Q} \end{cases} = \begin{cases} K\dfrac{D}{Q} + c_1 Q + h_1 \dfrac{Q}{2}, & \text{if } Q < \bar{Q} \\ K\dfrac{D}{Q} + c_2 Q + h_2 \dfrac{Q}{2}, & \text{if } Q \geq \bar{Q} \end{cases}$$

  - Then, the optimal order quantity is either the minimum of $C_U^1(Q)$ for $Q < \bar{Q}$, Q_1, or the minimum of $C_U^2(Q)$ for $Q > \bar{Q}$, Q_2, whichever gives the least cost.

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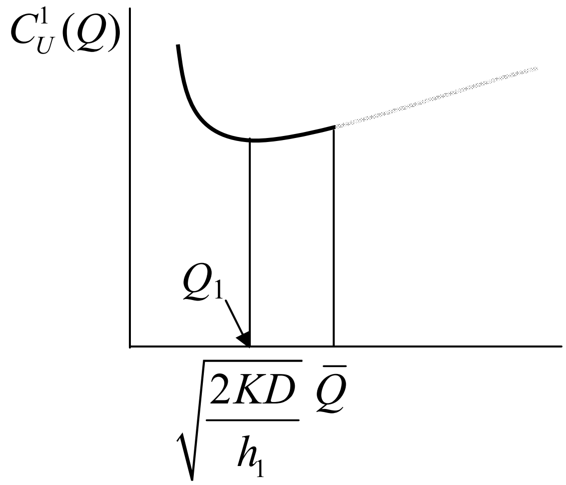
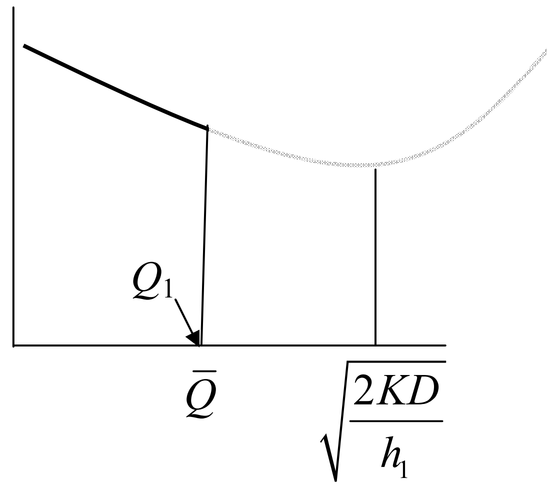
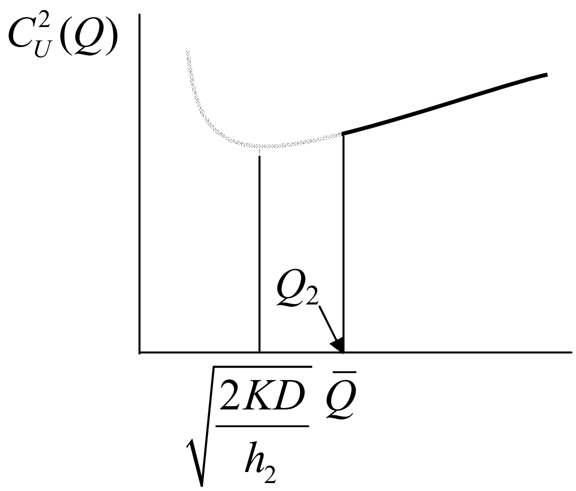
[1] The common form for the unit holding cost is $h = h_o + ic$, where h_0 is the "storage" cost and i is the cost of capital.

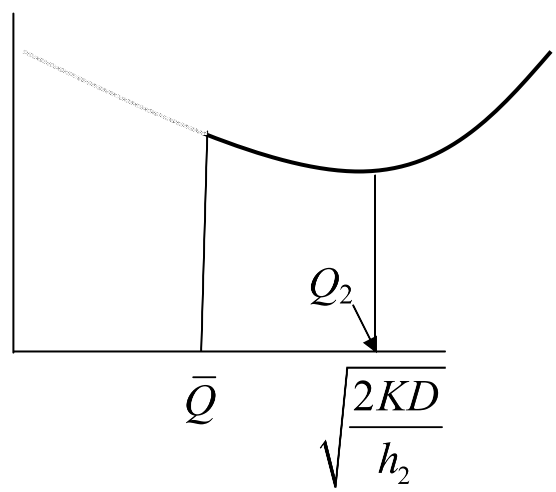
➢ Since $C_U^1(Q)$ is convex, then $Q_1 = \min(\bar{Q}, \sqrt{2KD/h_1})$





➢ Since $C_U^2(Q)$ is convex, then $Q_1 = \max(\bar{Q}, \sqrt{2KD/h_2})$





➢ Formally, the optimal order quantity is

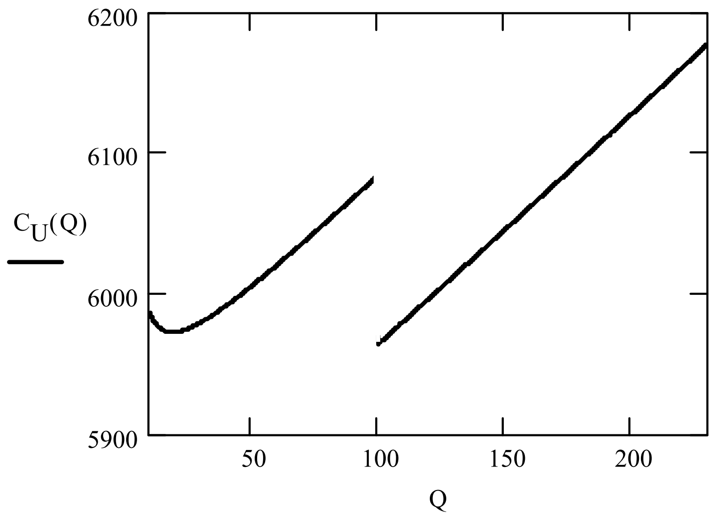
$$Q^* = \arg\min_{Q_1, Q_2}(C_U^1(Q_1), C_U^2(Q_2)) .$$

- **Example 4**
  - Because of convenience in manufacturing and shipping, the supplier of an item offers a 2% discount on any order of 100 units or higher. Demand for this item is estimated at 416 units per year. The unit variable cost is \$14.2, and the fixed cost is \$1.50. For this item storage costs are negligible, and the unit holding cost is 24% (the cost of capital) of the unit variable cost.
  - What are the optimal order quantity and annual cost for the item?
  - Here, K = \$1.5, c_1 = \$14.2, and $h_1 = 0.24 \times 14.2$ = \$3.41. In addition, $c_1 = 0.98 \times 14.2$ = \$13.92, and $h_2 = 0.24 \times 13.92$ = \$3.34.
  - Then, $Q_1 = \min(100, \sqrt{2 \times 1.5 \times 416 / 3.41}) \approx 19$ units, and

    $C_U^1(Q_1) = \sqrt{2 \times 1.5 \times 416 \times 3.41} + 14.2 \times 416 \approx \$5{,}972$

    $Q_2 = \max(100, \sqrt{2 \times 1.5 \times 416 / 3.34}) = 100$, and

    $C_U^2(Q_2) = 1.5 \times \frac{416}{100} + 13.92 \times 416 + 3.34 \times \frac{100}{2} \approx \$5{,}964$.
  - The optimal order quantity is therefore Q^* = 100, and the corresponding annual cost is \$5,964.
  - The annual cost as a function of Q is as shown below.



- **Time-varying demand – Wagner-Whitin model**
  - Suppose now that demand and costs are time variable.
  - Specifically, suppose the planning horizon is divided into n periods, numbered 1, 2,…, n.
  - Demand occurring at the beginning of period t is D_t (E.g., D_t can be obtained by a forecast based on last year demands).
  - Let c_t be the unit purchasing cost in period t.
  - Let K_t be the fixed setup cost in period t.
  - An order could be placed at the beginning of period t. If an order of size Q_t is placed, the ordering cost is $K_t + c_tQ_t$.
  - The deliver lead time of an order is zero.
  - The holding cost is proportional to *end of period inventory*. If period t ends with inventory level I_t, then the holding cost is h_tI_t , where h_t is the unit holding cost in period t.
  - Suppose (for now) that the initial inventory is zero.
  - No shortages are allowed.

- The objective in this model is to determine the ordering quantities in every period, Q_t, in a way that minimizes cost, and meets all demand.
- Such a time varying (dynamic) demand model is known as the Wagner-Whitin model.
- Wagner and Whitin (1958) proved two key properties of this model:
  1. In an optimal policy, an order is placed in period t only if the starting inventory in period t is zero.
  2. In an optimal policy, if an order is placed in period t, then the order size, Q_t, will cover the demand of one or more subsequent periods. That is,

$$Q_t \in \{D_t, D_t + D_{t+1}, D_t + D_{t+1} + D_{t+2}, \ldots, \sum_{i=t}^{n} D_i\} .$$

- These two properties allow the problem to be solved recursively according to the following *dynamic program*.
- Let C_t be the (minimum) cost of the best ordering policy from period t to period n, starting with zero inventory in period t (an order must be placed in period t).
- Then, C_t can be determined recursively as follows.

$$C_t = \min_{j=t,t+1,\ldots,n} \left\{ K_t + c_t \sum_{i=t}^{j-t} D_t + \sum_{i=t}^{j-t} h_i \sum_{j=i+1}^{j-t} D_t + C_{j+1} \right\},$$

where $C_{n+1} = 0$, and sums over empty sets are zero.

- The recursive equation determines how many subsequent periods' demand is ordered in period t.
- Note that the evaluation of C_1 allows determining the optimal order quantity.

- **Example 5**
  - A market survey conducted by a television manufacturer has indicated that the demand for television sets is seasonal. In particular, sales 0f 30,000 sets is forecast for the season Christmas (October to December), 20,000 for the winter slack season (January to March), 30,000 for the "new model" season (April to June), and 20,000 for the summer season (July to September). To meet demand for the respective seasons, a certain TV component must be available at the beginning of the season (1 component is required for each TV). For this component, the fixed and variable order costs are assumed to be $20,000 and $1 in all periods. The holding cost is also assumed constant at $0.2/unit.
  - What is the optimal production policy for the component?
  - Since all ordering quantities are multiples of 10 K, we can redefine the problem parameters in multiple of 10 K as follows: $D_1 = 3$, $D_2 = 2$, $D_3 = 3$, $D_4 = 2$,

$$c_t = 1,\ h_t = 0.2,\ K_t = 2,\ t = 1,2,3,4$$

- The dynamic programming recursive proceeds as follows
  - In period 4,

$$C_4 = K_4 + c_4 D_4 + C_5 = 2 + 1 \times 2 + 0 = 4$$

That is, if period 4 starts with zero inventory the best policy and (only alternative) is to place an order covering demand for period 4 at a cost of \$40 K.

  - In period 3,

$$\begin{aligned} C_3 &= \min\{K_3 + c_3 D_3 + C_4, \\ &\qquad K_3 + c_3(D_3 + D_4) + h_3 D_4 + C_5)\} \\ &= \min\{2 + 3 + 4, 2 + 5 + 0.2 \times 2 + 0\} = \min\{9, 7.4\} = 7.4 \end{aligned}$$

That is, if period 3 starts with zero inventory the best policy is to place an order covering demands for period 3 and 4 at a cost of \$74 K.

  - In period 2,

$$\begin{aligned} C_2 &= \min\{K_2 + c_2 D_2 + C_3, \\ &\qquad K_2 + c_2(D_2 + D_3) + h_2 D_3 + C_4, \\ &\qquad K_2 + c_2(D_2 + D_3 + D_4) + h_2(D_3 + D_4) + h_3 D_4 + C_5\} \\ &= \min\{2 + 2 + 7.4,\ 2 + 5 + 0.2 \times 3 + 4,\ 2 + 7 + 0.2 \times 5 + 0.2 \times 2 + 0\} \\ &= \min\{11.4, 11.6, 10.4\} = 10.4 \end{aligned}$$

That is, if period 2 starts with zero inventory the best policy is to place an order covering demands for periods 2, 3 and 4 at a cost of \$104 K.

- In period 1,

$$
\begin{aligned}
C_1 = \min\{ & K_1 + c_1 D_1 + C_2, \\
& K_1 + c_1(D_1 + D_2) + h_1 D_2 + C_3, \\
& K_1 + c_1(D_1 + D_2 + D_3) + h_1(D_2 + D_3) + h_2 D_3 + C_4 \\
& K_1 + c_1(D_1 + D_2 + D_3 + D_4) + h_1(D_2 + D_3 + D_4) + h_2(D_3 + D_4) + h_3 D_4 + C_5\} \\
= \min\{ & 2+3+10.4,\ 2+5+0.2\times 2+7.4,\ 2+8+0.2\times 5+0.2\times 3+4, \\
& 2+10+0.2\times 7+0.2\times 5+0.2\times 2+0\} \\
= \min\{ & 15.4, 14.8, 15.6, 14.8\} = 14.8
\end{aligned}
$$

That is, the optimal policy is one of two alternatives:

1. Produce 10 “units” (for all four periods) in period 1. Don’t produce in other periods.
2. In period 1, produce 5 units (for periods 1 and 2). Then, in period 3 produce 5 units (for periods 3 and 4). Don’t produce periods 2 and 4.

Both alternatives incurs a cost of \$148 K.