Chapter 18 Sensitivity Analysis and Expected Value Decisions

- **Sensitivity analysis**
  - Engineering economy estimates of parameters such as costs and other cash flow are only an approximation of reality.
  - The *realized* future value of a parameter will be generally different from its estimated value.
  - Sensitivity analysis attempts to measure the effect of this *uncertainty* (variability) in parameters estimates.
  - Sensitivity analysis identifies parameters that have the most impact on an economic decision.
  - That is, it identifies the parameters with the highest *sensitivity*.
  - Sensitivity of a parameter is the effect of changing the parameter value on the economic criteria such as PW value.

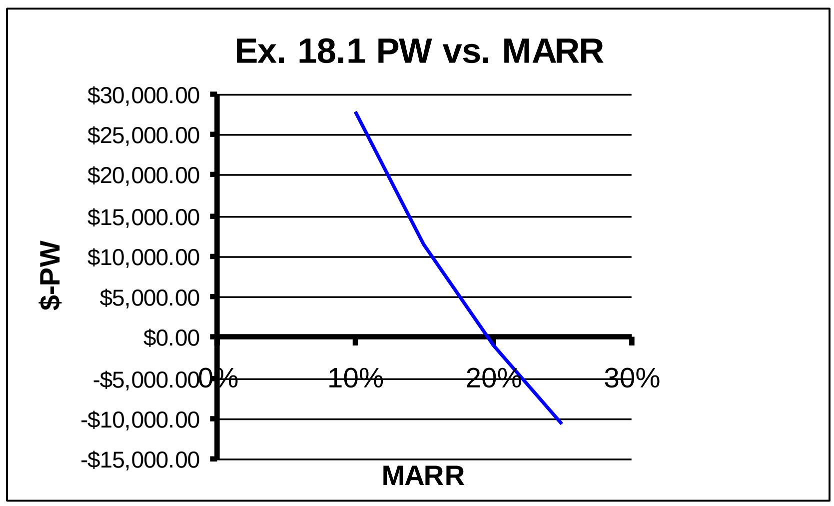
- **Graphical sensitivity analysis**
  - Sensitivity analysis is often done graphically by plotting the economic criteria as a function of a parameter.
  - The plot is over the range where the parameter is most likely to vary.
  - A flat plot indicates insensitivity.

- That is, the parameter has little effect on the economic decision. No need to get very precise estimates for its value.
- A highly variable plot indicates high sensitivity.
- That is, the parameter has a significant impact on economic decision. Estimating its value should be handled with care.

- **Example**
  - Example 18.1 gives PW vs. MARR for a project.

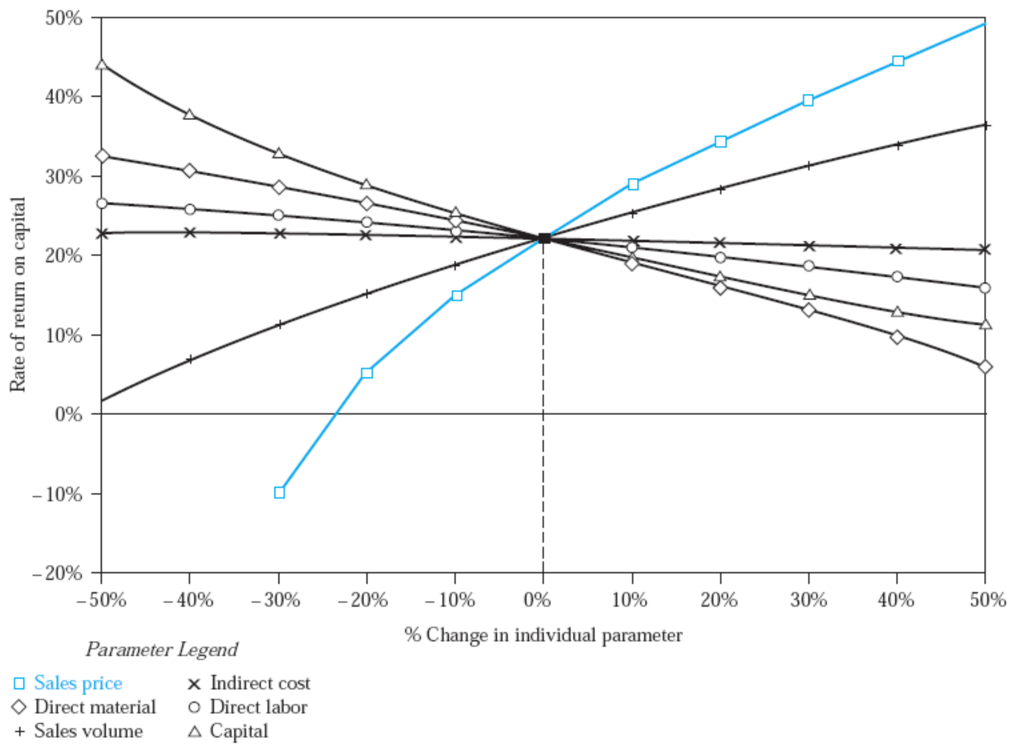
Year	Cflow
0	-$80,000
1	$25,000
2	$23,000
3	$21,000
4	$19,000
5	$17,000
6	$15,000
7	$13,000
8	$11,000
9	$9,000
10	$7,000

MARR	PW(i%
10%	$27,831.49
15%	$11,510.26
20%	-$962.36
25%	-$10,711.51



- In this example, PW is highly sensitive to MARR.

- **Sensitivity analysis with several parameters**
  - ➢ When several parameters are being analyzed for sensitivity, A *spider plot* can be used.
  - ➢ This is a plot of the economic criteria as a function of percent changes from the most likely estimates of parameters.
  - ➢ Example of spider plot (Fig 18.3).

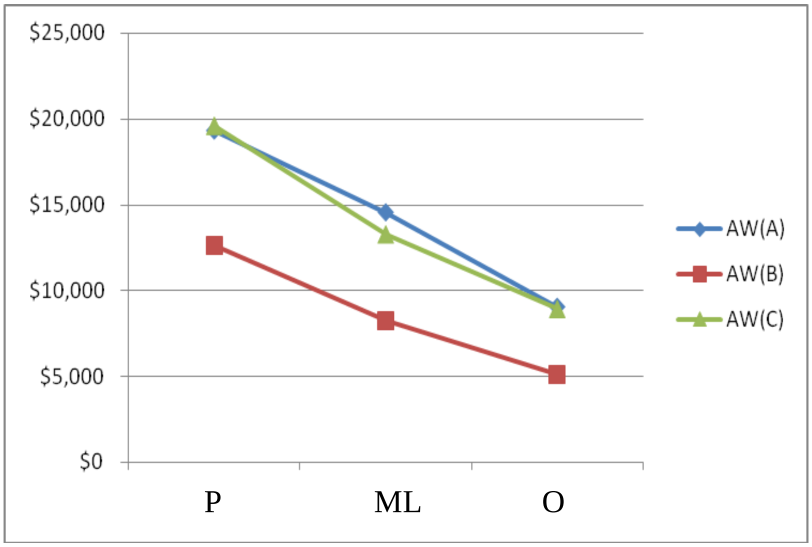


  - ➢ In this example, ROR is
    - o Insensitive to indirect cost (flat curve), and labor cost;
    - o Moderately sensitive to material cost and capital;
    - o Highly sensitive to sales volume and sales price.

- **Sensitivity analysis with three estimates**
  - A common approach is to base sensitivity analysis on three estimates for a parameter: *pessimistic*, *most likely*, and *optimistic* estimate.
  - Example 18.3 performs sensitivity analysis of annual cost for three alternatives based on three estimates of parameters.
  - In this example, parameters are assumed to change together for the pessimistic, most-likely and optimistic scenarios.
  - This is also known as *scenario analysis*.

Strategy	First Cost	SV	AOC	Life	AW
Alternative A					
Pessimistic	-$20,000	$0	-$11,000	3	-$19,327
Most likely	-$20,000	$0	-$9,000	5	-$14,548
Optimistic	-$20,000	$0	-$5,000	8	-$9,026
Alternative B					
Pessimistic	-15,000	500	-4,000	2	-$12,640
Most likely	-15,000	1,000	-3,500	4	-$8,229
Optimistic	-15,000	2,000	-2,000	7	-$5,089
Alternative C					
Pessimistic	-30,000	3,000	-8,000	3	-$19,601
Most likely	-30,000	3,000	-7,000	7	-$13,276
Optimistic	-30,000	3,000	-3,500	9	-$8,927

  - According to the plot below, alternative B is better than alternatives A and C under all scenarios.
  - Selecting Alternative B is a good choice.



- **Variability and probability**
  - When a cash flow (parameter) estimate is highly uncertain and the economic decision is highly sensitive to it, a *probabilistic* cash flow analysis is useful.
  - Probability of an event (here the cash flow taken on a value) is typically defined as the long run fraction of time where the event happens.
  - If the "event" repeats several times, then one can use historical date to estimate the probability. That is, probability is estimated as a *frequency*.
  - E.g., one can estimate that the probability that the energy cost in August exceeds $1,000 is 0.2, if this cost exceeds 1,000 in 20% of past years.

- However, in several situations, especially in one-time engineering projects, not enough historical data is available.
- E.g., for the construction of an exotic tower, the experienced civil engineer estimates that there is a 95% chance (i.e. a 0.95 probability) that the cost will not exceed $1 billion.
- This is a *subjective* estimate of probability.

- **Cash flows as random variables**
  - A random variable is a function that assigns real numbers to events (outcomes of an experiment).
  - E.g., the salvage value of a machine, *S*, is $1,500 if the market goes up (with probability 0.4) and $1,000 if the market goes down (with probability 0.6).
  - Then, *S* is a random variable with the following *probability distribution* $P\{S = 1500\} = 0.4$ and $P\{S = 1000\} = 0.6$.
  - If the random variable can take on a limited number of values. Then, this is a *discrete random variable*. E.g., the salvage value of the machine above.
  - If the random variable can take on an uncountable number of values. Then, this is a *continuous random variable*.
  - E.g., you may estimate that the monthly fuel consumption of your car is equal likely to be between $200 and $250. Then, this consumption is a continuous random variable.

- **Expectation of a random variable**
  - The *expectation* of a discrete random variable X is

$$E[X] = \sum_{x_i} x_i P\{X = x_i\}$$

  - E.g., for the machine above the expected salvage value is

$$E[S] = 1{,}500\times0.4 + 1{,}000\times0.6 = \$1{,}200.$$

  - The expectation of a random variable is the value obtained if the underlying experience is repeated for a number of times which is large enough and the resulting values are averaged.

- **Using expected value – Replacing cash flows with their averages**
  - Here one finds the expected value of an uncertain (random) cash flow, and then uses this value in the economic analysis, proceeding just like what we have been doing.
  - This, often used, approach is only an approximation. It is subject to the *flow of averages*.

- **Example of the flow of average**
  - E.g., if the weekly demand for an item selling at \$5 is equally likely to be 100 or 150. The stock level for this item at the beginning of the week is set at 125 units.
  - The average demand in this case in 125.
  - Replacing the demand by its average, the "expected revenue" is 125×5 = \$625.

- Note however, that there is a 50% chance that the demand will be 100, and the revenue will be 100×5 =$500.
- There is also a 50% chance that demand will be 150 which exceeds the stock level.
- Then, the revenue when demand is 150 is 125×5 = $625.
- So, the actual expected revenue is

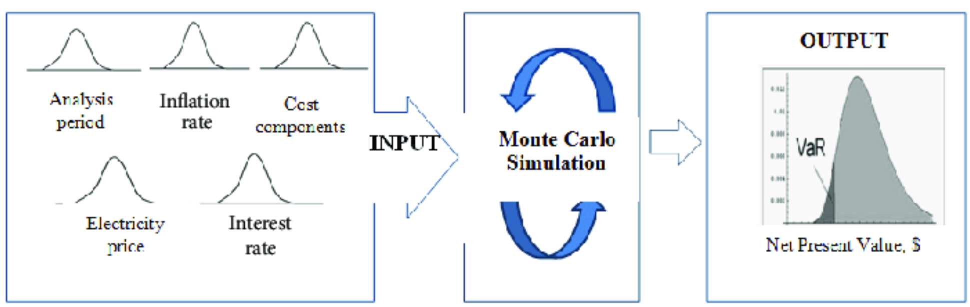
$$500\times0.5 + 625\times0.5 = \$562.50.$$

- **Using expected value – Treating cash flows a random variables and finding the expected PW**
  - Due to the flow of averages, a better approach for dealing with uncertainty in cash flows is to treat these as random variables and find the expected value, $E[PW]$ of the PW.
  - (This approach is more accurate but it often involves an involved use of *probability theory*. In INDE 303, you can learn more about this.)
  - Then, an alternative is considered economically viable if its $E[PW] \geq 0$.
  - When comparing several alternatives, the alternative with the highest $E[PW]$ is selected.

- **Measures of Variability and Analysis beyond E[PW]**
  - The *variance* of a discrete random variable X is

    $$\text{Var}[X] = E[(X - E[X])^2] = \sum_{x_i} x_i^2 P\{X = x_i\} - \left(E[X]\right)^2 .$$

  - The *standard deviation* of X is $\sigma[X] = \sqrt{\text{Var}[X]}$.
  - It measures the variability around the expectation.
  - An alternative (or an estimate) having a PW with a large variance should be handled with care.
  - In modern risk analysis, a common approach is to use *Monte Carlo simulation* to estimate the full probability distribution of PW based on the distribution of input parameters.



  - There are highly user-friendly tools, Excel add-in tools, for doing the Monte Carlo analysis, e.g. @Risk and Crystal Ball.
  - In addition, to E[PW] and var[PW], Monte Carlo simulation allows estimating the *Value at Risk* (VaR), which is the α-percentile of the distribution.
  - Often, α = 5%, and the VaR satisfies P{PW ≤ VaR} = 5%.
  - Having a PW with VaR > 0 is re-assuring, as it implies that there is only a 5% that a project will be economically invalid.