Chapter 12 Independent Projects with Budget Limitation

- **Basic problem (capital budgeting problem)**
  - An investor can choose from a set S of n independent projects.
  - At year 0, the investor has only b dollars to invest.
  - Suppose that each project $j \in S$ requires an initial investment of NCF_{j0}.
  - Let NCF_{jt} be the net cash flows of project j at year t.
  - Suppose that the investor can provide any capital needed beyond year 0.
  - The problem is to select the subset of projects, $A \subseteq S$ that maximizes the investor's fortune while meeting the budget constraint at year 0.
- **Solution by Total Enumeration**

1. Identify "feasible bundles" that satisfy the budget constraint.

   That is, find all $A \subseteq S$, such that $\sum_{j \in A} NCF_{j0} \le b$.

2. Find the PW of each of these "feasible" bundles. The PW of a bundle is the sum of PWs of projects within the bundle. That is, $PW_A = \sum_{j \in A} PW_j$.

3. Select the feasible bundle that gives maximum PW. That is, select A^* such that $PW_{A^*} = \max_{A \subset S} PW_A$.

- **Underlying assumption**
  - PW of each project is determined on its own life span. No LCM or study period approach is needed.
  - This is equivalent to assuming that positive cash flows of short life projects in a bundle are reinvested throughout the life of the longest life project at the MARR (Page 324, text.)

- **Solving the capital budgeting problem using ILP**
  - Solving the capital budgeting problem by hand using total enumeration can be tedious especially when the choice set S has many projects (2^n bundles should be considered).
  - Integer linear programming (ILP) is an optimization technique that can be used to solve this problem quickly using a computer.
  - In ILP, each project $j \in S$ is assigned a binary variable x_j. The variable x_j can take on two values 0 and 1.
  - If $x_j = 0$, Project j is not selected. If $x_j = 1$, Project j is selected.
  - The ILP "model" for the capital budgeting problem is

$$\max \quad Z = \sum_{j \in S} PW_j x_j$$

subject to

$$\sum_{j \in S} NCF_{j0} x_j \leq b \ .$$

- ➢ The ILP problem can be solved using several commercially available software (e.g., Microsoft Excel Solver).
- ➢ The solution to the ILP is denoted by $\mathbf{x}^* = (x_1^*, x_2^*, \ldots, x_n^*)$. A project j is selected if and only if $x_j^* = 1$.
- ➢ Note that the ILP formulation is flexible and can accommodate additional practical constraints.
- ➢ For example, suppose Projects 1 and 2 are mutually exclusive (i.e., if Project 1 is selected, or Project 2 is not selected, and vice versa). Then the following constraint is added,

$$x_1 + x_2 \leq 1 \ .$$

- ➢ A “precedence” relations which implies that Project 2 cannot be chosen unless Project 1 is chosen requires this constraint,

$$x_2 \leq x_1 \ .$$

- **Approximate (Heuristic) Capital Budgeting Solutions**
  - ➢ Approximate solutions to the capital budgeting ILP is obtained by ranking projects according to *benefit-cost ratio*.
  - ➢ This approximation method assumes that each project involves an initial outlay of cash (first cost) followed by a series of benefits.
  - ➢ The benefit-cost ratio for each project is the ratio of the *present value of benefits* to the initial cost.
  - ➢ The approximate solution is obtained by selecting the project with the highest benefit-cost ratio, then the project with the second highest ratio, and so on, until the budget is exhausted.