Chapter 7 Rate of Return Analysis: Single Alternative (1)

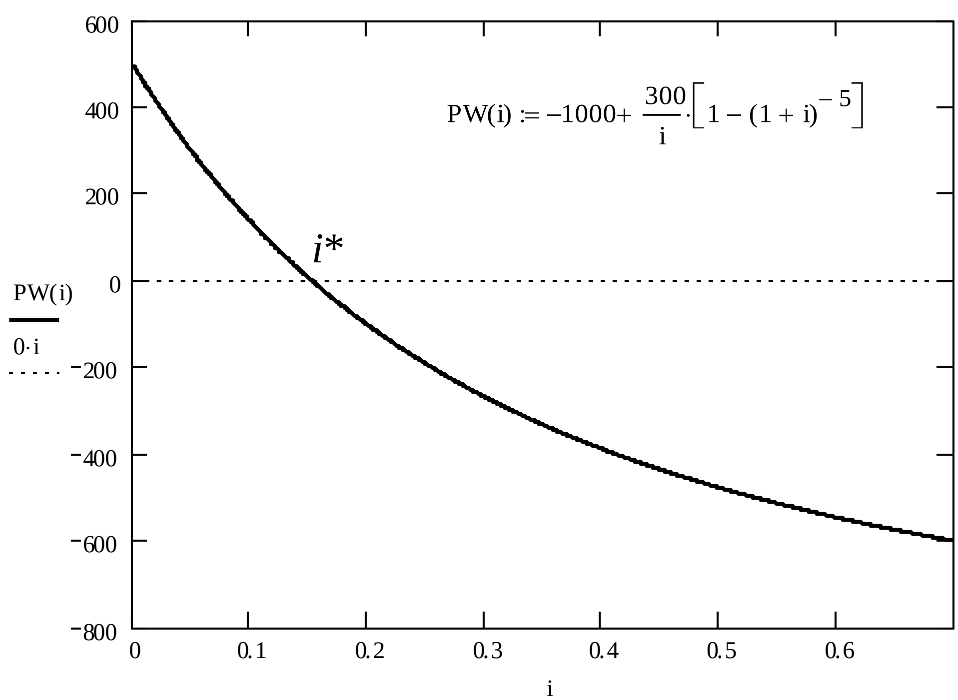
- **Introduction**
  - ➢ The rate of return (ROR) for an investment involving one single payment P and returning F after 1 year

$$\text{ROR} = i^* = F / P - 1 .$$

  - ➢ Alternatively, i^*, is given by the solution to the equation

$$-P + F / (1+i^*) = 0 \Leftrightarrow PW(i) = 0.$$

  - ➢ With more sophisticated projects involving several payments and receipts, the same equation, $PW(i) = 0$, gives the ROR.
  - ➢ E.g., for a \$1,000 investment that pays \$300 per year over 5 years, the ROR is 15.2%, as shown in the graph below.



- **Definition: What is the ROR?**
  - ROR is the rate of interest paid on the *unrecovered balance* of an investment, so that the final receipt brings the balance to exactly 0 with interest considerations.

- **Facts: What the ROR is not?**
  - ROR is *not* the rate of interest earned on the original investment amount.[1]
  - The simple example on the previous page of the $1,000 returning $300/year over five years demonstrates this clearly.
  - First, the ROR is not 30%. It is 15.2%!
  - Second, the amount of interest earned in each of the five years is *not* (15.2%)(1,000) = $152. This is in Year 1 only.
  - In later years, the interest (on unrecovered balance) is less,

Year	Interest	Payment	Balance
0	--	--	$ 1,000
1	$ 152	$ 300	$ 852
2	$ 130	$ 300	$ 682
3	$ 104	$ 300	$ 486
4	$ 74	$ 300	$ 260
5	$ 40	$ 300	$ (0)

- **Deciding on an alternative based on ROR**
  - If $i^* \geq$ MARR, accept alternative.
  - If $i^* <$ MARR, reject alternative.

[1] *Installment financing* is paying interest based on the loan principal (initial balance). The rate that lenders provide to promote such loans is not their true ROR. (It may put the borrower at a great disadvantage.)

- **Range for *i** and relation to PW**
  - $-100\% < i^* \le +\infty$.
  - $i^* \ge \text{MARR} \Leftrightarrow \text{PW(MARR)} \ge 0$.[2]

- **ROR calculation using a PW or AW equation**
  - Set PW(i) = 0 or AW(i) = 0, this leads to
    - $-PW_D(i^*) + PW_R(i^*) = 0 \Leftrightarrow PW_D(i^*) = PW_R(i^*)$,
    - $-AW_D(i^*) + AW_R(i^*) = 0 \Leftrightarrow AW_D(i^*) = AW_R(i^*)$,

    where "*D*" and "*R*" denote disbursements and receipts.
  - This usually involves finding the root(s) of an n^{th} degree polynomial where n is the project life.
  - By hand, ROR is found by solving one of the above equations by trial and error.
  - Using a computer package, such as Excel, ROR can be usually found easily.
  - To solve the equation quickly, use a good starting solution.

- **Starting ROR Solution**
  - A good starting value for ROR is found as follows:
    - "Convert" all disbursements into either a single or a uniform amount without considering time value of money. (This is a rough approximation.)
    - "Convert" all receipts into either a single value or a uniform series.

[2] This assumes that the project cash flows admit a unique ROR solution. More on this below.

- Solve the resulting PW = 0 equation which will be of the form $PW_D = PW_R \times Factor(i^*)$
- The starting value is an approximation to ROR.
- Another good way to get a starting solution, especially when using a computer, is to plot PW (or AW) versus *i* .
- This allows eyeballing a good starting value of *i**.

- **Solution by computer (Excel)**
  - In Excel, if cash flows, over *n* years, involve an initial payment, *P*, followed by a uniform series of receipts, *A*, and then a final (salvage) value, *F*.
  - Then, ROR is given by the function *RATE*(*n, A, P, F*). (Note that *P* should have a negative value.)
  - If the cash flows do not follow this particular pattern, an initial "guess" value should be determined.
  - Then, cash flows are to be inputted in detail (say in the range first_cell:last_cell).
  - Then, ROR is given by *IRR(first_cell:last_cell, guess)*.

- **Advantages of using ROR**
  - No immediate need to estimate MARR.
  - It is somewhat intuitive and easy to understand.
  - Investors like it.

- **Disadvantages of using ROR**
  - For some types of cash flows the ROR method can be computationally difficult.
  - Some cash flows will result in multiple ROR solutions. This raises questions as to which, if any, solution to use.
  - Special procedure is required when comparing multiple alternatives (Chapter 8).

- **Checking whether there are multiple ROR solutions**
  - There are two tests to check whether a cash flow series could have multiple ROR solutions
  - The tests are based on the idea that the ROR is one of the roots of the polynomial $PW(c) = \sum_{t=0}^{n} F_t c^t$, where $c = 1/(1+i)$ and F_t is the cash flow at time t.
  - *Descartes' rule of signs*.

    The maximum number of roots (ROR solutions) of a cash flow series is equal to the number of sign changes.
  - *Norstrom's criterion*.

    Let $S_t = \sum_{r=0}^{t} F_r$ be the cumulative cash flows at time t. The cash flow series has a unique ROR solution if
    - $S_0 < 0$;
    - The series S_0, S_1, …, S_n, changes sign only once (from minus to plus).

- When the two tests indicate that multiple ROR values *may* exist the next step is to plot PW(i) vs. i to find out how many RORs really exist.

- **Conventional / unconventional cash flow series**
  - A cash flow series is said to be conventional if it changes sign only once (from minus to plus).
  - Otherwise, the series is called unconventional.
  - Fortunately, conventional cash flows admit a unique ROR solution (based on Descartes' rule).
  - Unconventional cash flows may admit multiple RORs.
  - There is no universally accepted way for analyzing these cash flows.
  - Some multi-ROR analysis methods are based on assuming a *reinvestment rate* for positive cash flows.
  - *In this course, we shy away from analyzing cash flows with multiple ROR solutions.*[3]
- **Ex. of unconventional investment with multiple ROR sols.**
  - Example 7.4 (text) considers a project with the following cash flows in $K, (2, -0.5, -8.1, 6.8).
  - Obviously, this is an unconventional investment.
  - Descartes' rule indicates a maximum of two roots.
  - Norstrom's criterion does not apply as $S_0 = 2 > 0$.

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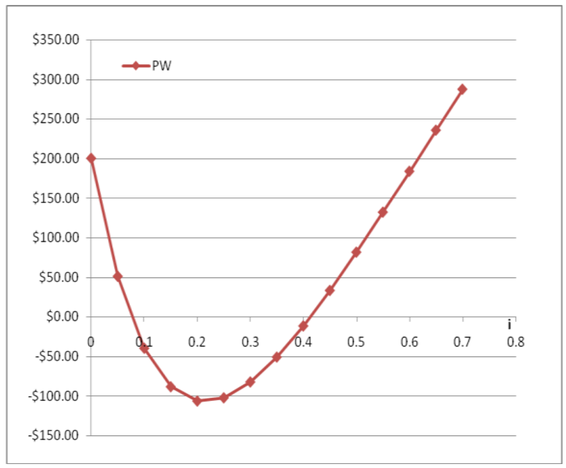
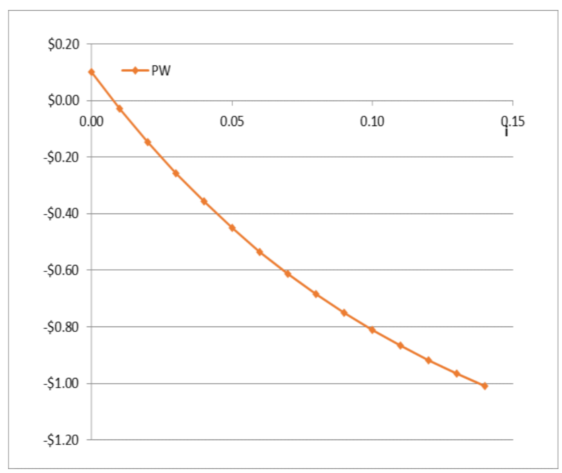
[3] After some research, it is this instructor's modest belief that the ROR methods is meant for conventional investments only. Unconventional investments, for what they worth, can be analyzed via the PW method.

- In fact, there are two ROR solutions here.
- The two values are 7.47% and 41.35% (found using the Excel IRR() function.)
- Here it is not obvious which value is the right ROR.



- **Ex. of unconventional investment with a single ROR sol.**

Year, t	F_t	ROR
0	0	0.77%
1	-$2.00	
2	-$2.00	
3	$2.50	
4	-$0.50	
5	$0.60	
6	$0.50	
7	$0.40	
8	$0.30	
9	$0.20	
10	$0.10	



- **Conclusion about the space of investments**