Chapter 4 Nominal and Effective Interest Rates

- **Illustrative Example**
  - You placed \$100 in a saving account for one year at an interest rate of 1% *per month*.
  - Calculate the amount of interest and annual interest rate.
    - $F = P(1+i)^n = 100 \times 1.01^{12} = \$112.68$. The interest earned is $112.68 - 100 = \$12.68$.
    - The annual interest rate is 12.68/100 = 12.68%.
  - We say that the *effective* annual interest rate is 12.68%.
  - Or, <u>the interest rate is 12% per year, compounded monthly</u>.
  - That is, the effective annual interest that corresponds to a 12% *nominal* annual interest, *compounded* monthly is 12.68%.

- **Nominal interest rate**
  - A nominal interest rate is an interest rate that does not include any consideration of compounding.
  - Means “in name only”, “not the true, effective rate.” E.g.,
  - 12% per year, compounded monthly
    - 12% is NOT the true effective rate (per year)
    - 12% represents the nominal rate
  - Nominal interest rate is commonly referred to as “APR” (annual percentage rate).

- **Effective interest rate**
  - Effective interest rate is the actual rate that applies for a stated period of time.
  - It takes into account the effect compounding of interest
  - Effective interest is stated in the following form:

    r (per year), compounded every CP.
  - It involves two parameters
    - The annual nominal rate r.
    - The compounding period, CP, the time where interest applies
    - E.g.,
      - Daily compounding, CP = 1 day = 1/365 year.
      - Weekly compounding, CP = 1 week =1/52 year.
      - Monthly compounding, CP = 1 month = 1/12 year.
      - Quarterly compounding, CP = 3 months = 1/4 year.
      - Semiannual compounding, CP = 6 months = 1/2 year.
  - The effective rate is called APY (annual percentage yield).

- **Factors under *m*-time-a-year compounding**
  - Under compounding over a period $CP = 1/m$ year (e.g., CP = 1/12 year = 1 month), and at a nominal interest rate r, a present current P is equivalent after k periods (e.g. months) to

$$F = P(1+r/m)^k \Rightarrow (\mathrm{P/F}, r, m, k) = (1+r/m)^k .$$

  - Similarly, F dollars after k periods are equivalent to

$$P = F / (1+r/m)^k \Rightarrow (\mathrm{F/P}, r, m, k) = 1 / (1+r/m)^k .$$

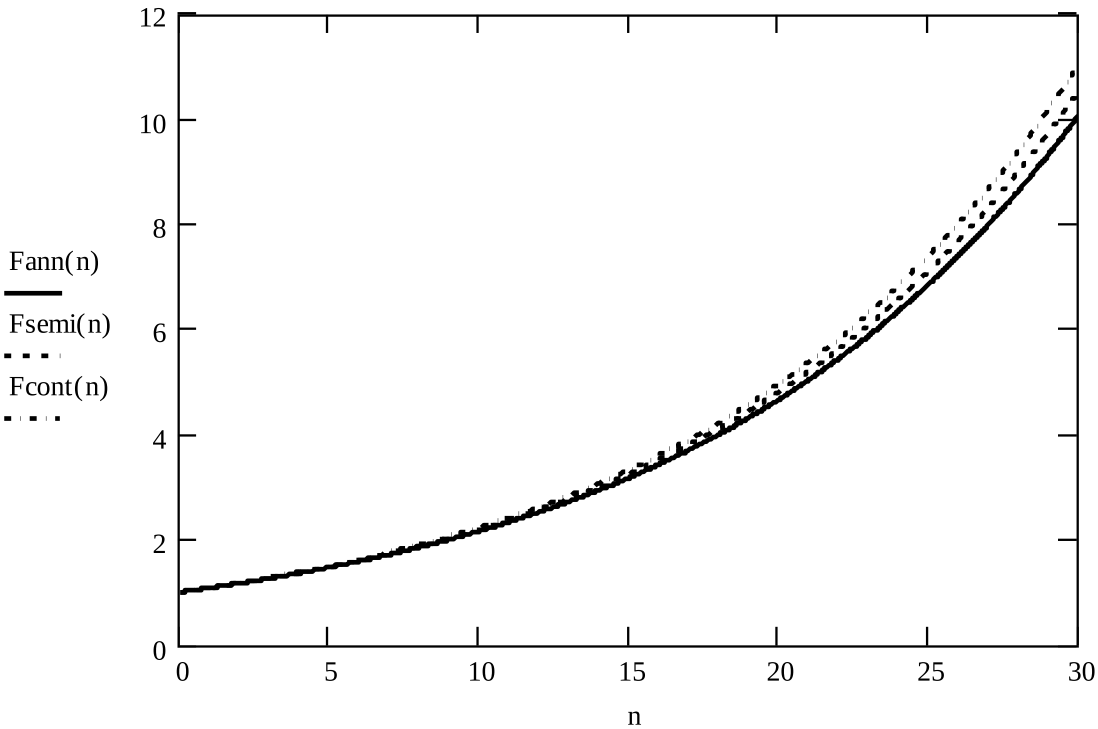
- **Computing the effective interest rate**
  - Note that the effective interest rate per *CP* is r / m, where m = 1/*CP*, with *CP* given in fraction of a year, is the number of times interest is compounded per year.
  - E.g., with monthly compounding, m = 12, and a nominal rate of 12% translates into an effective monthly rate of 1% .
  - With semiannual compounding, m =2, and a nominal rate of 12% translates into an effective semiannual rate of 6% .
  - Then, \$1 is equivalent to $(1+r/m)^m$ after 1 year.
  - The effective annual rate is such that $1+i=(1+r/m)^m$. I.e.,

$$i=\left(1+\frac{r}{m}\right)^m-1.$$

- **Continuous compounding**
  - If the compounding period, *CP*, is too small, *CP*$\to$ 0, the number of compounding times gets too large, $m\to\infty$.
  - This situation is known as “continuous compounding.”
  - By noting that there are mt compoundings over a time t expressed in years, the F/P factor over time t, under continuous compounding, is $\lim_{m\to\infty}(1+r/m)^{mt}=e^{rt}$
  - Similarly, under continuous compounding, the effective annual rate is $i=e^r-1$.
  - Continuous compounding is often assumed in quantitative finance as it simplifies the analysis.

- **Example: $1 invested at a nominal rate of 8%**

Year	***m* = 1**	***m* = 2**	***m* = 4**	***m* = 12**	***m* = ∞**
1	1.080	1.082	1.082	1.083	1.083
2	1.166	1.170	1.172	1.173	1.174
3	1.260	1.265	1.268	1.270	1.271
4	1.360	1.369	1.373	1.376	1.377
5	1.469	1.480	1.486	1.490	1.492
6	1.587	1.601	1.608	1.614	1.616
7	1.714	1.732	1.741	1.747	1.751
8	1.851	1.873	1.885	1.892	1.896
9	1.999	2.026	2.040	2.050	2.054
10	2.159	2.191	2.208	2.220	2.226



- **Interest rate that varies with time**
  - In practice, interest rate may vary from one period to the other.
  - In particular, it is often *expected* that the interest rate will *increase* with time.
  - If the interest rates in periods, 1, …, n are i_1, …, i_n. Then, the future worth after n periods, F, of a present amount P is

$$F = P(1+i_1)(1+i_2)\ldots(1+i_n) = P\prod_{t=1}^{n}(1+i_t)\,.$$

  - The rates i_1, …, i_n are known as *short rates*.
  - The short rate i_t represents the expected 1-year rate after t years.